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A comparative study of spreading of novel corona virus disease by ussing fractional order modified SEIR model

Hussam Alrabaiah [a,b,*], **Muhammad Arfan** [c], **Kamal Shah** [c], **Ibrahim Mahariq** [d,e], **Aman Ullah** [c]

[a] *College of Engineering, Al Ain University, Al Ain, United Arab Emirates*
[b] *Department of Mathematics, Tafila Technical University, Tafila, Jordan*
[c] *Department of Mathematics, University of Malakand, Chakdara Dir (Lower), Khyber Pakhtunkhawa, Pakistan*
[d] *College of Engineering and Technology, American University of the Middle East, Kuwait*
[e] *Department of Electrical and Electronics Engineering, University of Turkish Aeronautical Association, Ankara, Turkey*

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Abstract In this research work, a non-linear dynamical modified SEIR model of the recent pandemic, due to Coronavirus-19 disease (COVID-19) for different countries like Malaysia and Pakistan, is considered under nonsingular fractional order derivative. For this model, some qualitative results, existence theory, and numerical solution are studied by using fixed point approach and fractional Adams-Bashforth method. The results are simulated corresponding to some real data of various fractional order by using Matlab. Hence, the suitability of the considered COVID-19 model for the current outbreak in two different countries Malaysia and Pakistan are shown by simulation.

1. Introduction

Here, we notify that in recent time a threatful outbreak, which has been originated from China is spreading throughout the world very rapidly. Thousands of people have been facing death due to this disease. The outbreak of a deadly and highly infected virus of the present era is a corona virus and it is identified in the Wuhan (Chinese city) on December 31, 2019 [1,2]. Since then, it has killed over 0.96 million people, while the infected people are more than 31.2 millions in more than 180 countries. The history of this virus traced back to 1965, when Tyrrell and Bynoe have identified and they passaged a virus named B814 [3]. This virus is found in human embryonic tracheal organ cultures acquired from the respiratory tract of an adult [4].

Most of the researchers and policy makers are struggling to control the disease from further spreading. One big factor of the spreading of this disease is immigration of infected people from a place to a place which effect more people and hence

* Corresponding author at: College of Engineering, Al Ain University, Al Ain, United Arab Emirates.
E-mail address: hussam89@yahoo.com (H. Alrabaiah).

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causing the spread of this disease. Therefore, on international level, many countries of the world have banned air traffic for some time and also have announced lock-down in cities so that some precautionary measure should be taken to reduce maximum loss of human lives. Also, most countries is trying to reduced the unnecessary traveling of people in order to reduce the cases of infection in their countries [5]. Scientists and researchers are working hard to investigate the cure or vaccine for the aforesaid outbreak, so that in future such a pandemic may be controlled.

Investigating properly about the pandemic, plays useful role in controlling of the disease in a society. Implementation of a suitable strategy against the disease transmission is another challenge. From medical engineering point of view, mathematical modeling approach is one of the key tool in order to handle these infectious diseases. Mathematical models have been established for different disease in history, for study, we refer [6–9]. Similarly the mentioned outbreak has been reported in large numbers of articles, reports, monographs, etc, (for detail see [1,2,10–17]). The mathematical models are mostly differential and integral equations of integer order (IDEs). However, for the last few decades, the non-integer order differential equations (FDEs) can be used to formulate real phenomena with greater degree of precision and accuracy. Further, their applications can be found in different areas of physical and medical science, like engineering, economics, control theory, finance and in epidemiology. Modern calculus is the generalization of classical integer-order calculus. The increasing interest of using FDEs in modeling of real world problems is due to its various properties which are not found in IDEs. In contrast of IDEs which are local in nature, the FDEs are non-local and possesses the memory effects which make it more superior then IDEs. It is also because, in many situations the future state of the model depends not only upon the current state but also on the previous history[18–21]. These features enables FDEs to model the phenomena having not only the non-Gaussian but also for non-Markovian behavior. Further, the classical IDEs are unable to provide the information in between two different integer values. Various type of fractional-order operators were introduced in existing literature to over come such limitations of integer-order derivatives. The applications of these fractional operators can be found in various fields. The interesting area of research in recent time is mostly devoted to investigate biological models of infectious diseases. Many investigation about the mathematical models are devoted to study stability theory, existence results and optimization, we refer few as [22–25].

Our fractional dynamical model is taken from the non linear dynamical system in which population remains constant, i.e. birth and natural deaths have the same ratio[26]. The recovery rate $R(t)$ and death rate are time-dependant. The model also assumes that susceptible individuals $S(t)$ are contagious upon coming into contact with infected $I(t)$ individuals not detected. It is assumed, however,that the infected are all quarantined $Q(t)$ and that they do not have contact with susceptible individuals $S(t)$. In turn,the susceptible population can be protected by confinement by moving to the protected population compartment $C(t)$. This assumption on $Q(t)$ means that hospital infections are not considered under this framework, therefore resulting in a potential underestimation of the real contagion extent. However, chose this option in an attempt to be conservative and the results should be interpreted as the current best-case scenario. It is assumed also that the protected population does not have contact with the infected individuals and therefore cannot be infected. The remaining individuals are exposed $E(t)$, infected $I(t)$, individuals whose deaths occur are represented by $D(t)$. The COVID-19 dynamics in integer form is modelled by the following equations system in [26] as

$$\begin{cases} \mathcal{D}_t(S(t)) = \mu N_T + \tau C(t) - \alpha S(t) - \beta \frac{S(t)I(t)}{N_T} - \mu S(t), \\ \mathcal{D}_t(E(t)) = -\gamma E(t) + \beta \frac{S(T)I(t)}{N_T} - \mu E(t), \\ \mathcal{D}_t(I(t)) = \gamma E(t) - \delta I(t) - \mu S(t) - \mu I(t), \\ \mathcal{D}_t(Q(t)) = \delta I(t) - \lambda(t)Q(t) - k(t)Q(t) - \mu Q(t), \\ \mathcal{D}_t(R(t)) = \lambda(t)Q(t) - k(t)Q(t) - \mu R(t), \\ \mathcal{D}_t(D(t)) = k(t)Q(t), \\ \mathcal{D}_t(C(t)) = \alpha S(t) - \mu C(t) - \tau C(t), \\ S(0) = S_0, \quad E(0) = E_0, \quad I(0) = I_0, \quad Q(0) = Q_0, \\ R(0) = R_0, \ D(0) = D_0, \quad C(0) = C_0. \end{cases} \tag{1}$$

where

$$k(t) = k_0 \exp(-k_1 t)$$

and

$$\lambda(t) = \lambda_0(1 - \exp(\lambda_1 t)).$$

The detail of parameters used in the model (1) with complete descriptions are given in Table 1. In this article we will take (1) in fractional differential form under Atangana-Baleanu-Caputo Derivative with fractional order $0 < s \leq 1$ as

$$\begin{cases} {}^{\mathbb{ABC}}\mathcal{D}_t^s(S(t)) = \mu N_T + \tau C(t) - \alpha S(t) - \beta \frac{S(T)I(t)}{N_T} - \mu S(t), \\ {}^{\mathbb{ABC}}\mathcal{D}_t^s(E(t)) = -\gamma E(t) + \beta \frac{S(T)I(t)}{N_T} - \mu E(t), \\ {}^{\mathbb{ABC}}\mathcal{D}_t^s(I(t)) = \gamma E(t) - \delta I(t) - \mu S(t) - \mu I(t), \\ {}^{\mathbb{ABC}}\mathcal{D}_t^s(Q(t)) = \delta I(t) - \lambda(t)Q(t) - k(t)Q(t) - \mu Q(t), \\ {}^{\mathbb{ABC}}\mathcal{D}_t^s(R(t)) = \lambda(t)Q(t) - k(t)Q(t) - \mu R(t), \\ {}^{\mathbb{ABC}}\mathcal{D}_t^s(D(t)) = k(t)Q(t), \\ {}^{\mathbb{ABC}}\mathcal{D}_t^s(C(t)) = \alpha S(t) - \mu C(t) - \tau C(t), \\ \quad S(0) = S_0, \quad E(0) = E_0, \quad I(0) = I_0, \quad Q(0) = Q_0, \\ R(0) = R_0, \ D(0) = D_0, \ C(0) = C_0. \end{cases} \tag{2}$$

Table 1 Description of the parameters used in model (1).

Notation	Parameters description
μ	Natural birth rate = Natural death rate($\frac{1}{80\times 365}$)
τ	Length of protection by confinement($\frac{1}{30}$)
α	Protection rate
β	Infection rate
γ	Incubation rate
δ	Quarantine rate
$\lambda(t)$	Recovery rate
$k(t)$	Mortality rate by the virous
k_0	Fitted Coefficient
k_1	Fitted Coefficient
λ_0	Fitted Coefficient
λ_1	Fitted Coefficent

For the last few decades, it is noted that non-integer-order differential equations (FDEs) can be apply to formulate real phenomena with greater degree of precision and accuracy. In eighteenth century when "Reimann and Liouville", Euler and Fourier provided useful results in classical calculus. Because of their contribution the area of modern calculus was also established and some good research has been carried out later on. This is due to lots of applications of modern calculus in the filed of mathematical modeling, where several hereditary concepts and memory process cannot be explained clearly by classical calculus. Because fractional calculus in which include classical calculus is a special case has greater degree of freedom in differential operator as compared to integer differential operator which is local in nature. The important applications of the said calculus may be titled in [18–21,27–30]. Therefore researchers and scientists have given very much interest in discussion of fractional derivatives and integrals. In fact fractional derivative is a definite integral which geometrically interpret the accumulation of the whole function or the whole spectrum which globalize it. On the other hand ordinary derivative is a special case of the fractional order. Analysis of differential equations for qualitative study, numerical and optimization, significant contribution has been made by researchers, we refer few as [31–37]. It is also remarkable that fractional differential operators have been defined by number of ways. It is well known fact that definite integral has no regular kernel, therefore both type of kernel have been involved in various definitions. One of the important definition which has very recently attracted the attention is the $\mathbb{ABC}$ derivative introduced by Atangana-Baleanu and Caputo [38] in 2016. The fractal-fractional derivatives exhibit the singular kernel by nonsingular kernel and therefore were greatly studied in [39–46,56–59]. Now the question how to solve these problems. In this regards plenty of methods available in literature which has been applied to the old definitions of fractional derivative. For instance, to handle nonlinear problems analytically, famous decomposition and homotopy methods were increasingly used (see [9,47,48]). For numerical purpose in simulation usually Runge Kutta methods were used in large number for dealing of mathematical modeling. Here for numerical simulation we will use fractional AB method for numerical simulation. The mentioned method is simple two step technique and more powerful than Euler and Taylor's and RK methods. The concerned method is powerful as well as rapidly converging and stable, (for detail see [49,50]).

2. Basic results

Definition 2.1. The $\mathbb{ABC}$ fractional differentiation of a function $\Omega(t)$ having the condition $\Omega(t) \in \mathcal{H}^1(0,\tau)$ is given by

$$^{\mathbb{ABC}}\mathbf{D}_0^s\Omega(t) = \frac{\mathbb{ABC}(s)}{1-s}\int_0^t \frac{d}{dz}\Omega(z)\kappa_s\left[\frac{-s}{1-s}(t-z)^s\right]dz. \tag{3}$$

We find that if we replace $\kappa_s\left[\frac{-s}{1-s}(t-z)^s\right]$ by $\kappa_1 = \exp\left[\frac{-s}{1-s}(t-z)\right]$, then we get the so-called Caputo-Fabrizo differential operator. Further it is to be mention that

$$^{\mathbb{ABC}}\mathbf{D}^r[Constant] = 0.$$

Here $\mathbb{ABC}(s)$ is known as normalization function which is defined as $\mathbb{ABC}(0) = \mathbb{ABC}(1) = 1$. Also κ_s stands for famous special function called Mittag-Leffler which is a generalization of the exponential function [28–30].

Definition 2.2. Let $\Omega \in L[0,T]$, then the corresponding integral in $\mathbb{ABC}$ sense is given by

$$^{\mathbb{ABC}}\mathbf{I}_0^s\Omega(t) = \frac{1-s}{\mathbb{ABC}(s)}\Omega(t) + \frac{s}{\mathbb{ABC}(s)\Gamma(s)} \times \int_0^t (t-z)^{s-1}\Omega(z)dz. \tag{4}$$

Lemma 2.3 (*See Proposition 3 in [51]*). *The solution of the given problem for* $0 < s < 1$

$$\begin{aligned} ^{\mathbb{ABC}}\mathbf{D}_0^s\Omega(t) &= Y(t), \ t \in [0,T], \\ \Omega(0) &= \Omega_0 \end{aligned}$$

is provided by

$$\Omega(t) = \Omega_0 + \frac{(1-s)}{\mathbb{ABC}(s)}Y(t) + \frac{s}{\Gamma(s)\mathbb{ABC}(s)}\int_0^t (t-z)^{s-1}Y(z)dz.$$

Note: For the qualitative analysis, let $0 \leqslant t \leqslant T < \infty$, we define Banach space

$$\mathbf{Z} = \mathbf{Y} = C\big([0,T] \times R^7, R\big),$$

where $\mathbf{Y} = C[0,T]$ under the norm

$$\|W\| = \|\Omega\| = \sup_{t\in[0,T]}[|S(t)| + |E(t)| + |I(t)| + |Q(t)| + |R(t)| + |D(t)| + |C(t)|].$$

The following fixed point theorem will be used to proceed in our main results.

Theorem 2.4 [52]. *Let* $\mathbf{A}$ *be a convex subset of* $\mathbf{Z}$*and assume that* $\mathbf{F}, \mathbf{G}$ *are two operators with*

1. $\mathbf{F}w + \mathbf{G}w \in \mathbf{A}$ *for every* $w \in \mathbf{A}$*;*
2. $\mathbf{F}$ *is contraction;*
3. $\mathbf{G}$ *is continuous and compact.*

Then the operator equation $\mathbf{F}w + \mathbf{G}w = w$ *has at least one solution.*

3. Qualitative analysis of the considered model

It is of great importance to ask weather a dynamical problem we investigate exist really or not. This question is answered by fixed point theory. Here we analyze the concerned need for our considered problem (2) in this part of the paper. Regarding to the aforesaid need, we express the right sides of model (2) as

$$\begin{cases} \mathbb{G}_1(S(t),E(t),I(t),Q(t),R(t),D(t),C(t)) = \mu N_T + \tau C(t) - \alpha S(t) - \beta \frac{S(T)I(t)}{N_T} - \mu S(t), \\ \mathbb{G}_2(S(t),E(t),I(t),Q(t),R(t),D(t),C(t)) = -\gamma E(t) + \beta \frac{S(T)I(t)}{N_T} - \mu E(t), \\ \mathbb{G}_3(S(t),E(t),I(t),Q(t),R(t),D(t),C(t)) = \gamma E(t) - \delta I(t) - \mu S(t) - \mu I(t), \\ \mathbb{G}_4(S(t),E(t),I(t),Q(t),R(t),D(t),C(t)) = \delta I(t) - \lambda(t)Q(t) - k(t)Q(t) - \mu Q(t), \\ \mathbb{G}_5(S(t),E(t),I(t),Q(t),R(t),D(t),C(t)) = \lambda(t)Q(t) - k(t)Q(t) - \mu R(t), \\ \mathbb{G}_6(S(t),E(t),I(t),Q(t),R(t),D(t),C(t)) = k(t)Q(t), \\ \mathbb{G}_7(S(t),E(t),I(t),Q(t),R(t),D(t),C(t)) = \alpha S(t) - \mu C(t) - \tau C(t), \\ S(0) = S_0, \quad E(0) = E_0, \quad I(0) = I_0, \quad Q(0) = Q_0, \quad R(0) = R_0, \quad D(0) = D_0, \quad C(0) = C_0. \end{cases} \tag{5}$$

We considered our system as by using (5)

$$\begin{aligned} {}^{\mathbb{ABC}}\mathbf{D}^{s}_{+0}\mathcal{Z}(t) = &\quad \Upsilon(t,\mathcal{Z}(t)), \ t \in [0,\tau], \ 0 < s \le 1, \\ \mathcal{Z}(0) = &\quad \mathcal{Z}_0. \end{aligned} \tag{6}$$

Taking integration in sense of $\mathbb{ABC}$, we get

$$\begin{aligned} \mathcal{Z}(t) = \mathcal{Z}_0(t) + \frac{(1-s)}{\mathbb{ABC}(s)}[\Upsilon(t,\mathcal{Z}(t))] \\ + \frac{s}{\mathbb{ABC}(s)\Gamma(s)}\int_0^t (t-y)^{s-1}\Upsilon(y,\mathcal{Z}(y))dy, \end{aligned} \tag{7}$$

such that

$$\mathcal{Z}(t) = \begin{cases} S(t) \\ E(t) \\ I(t) \\ Q(t) \\ R(t) \\ D(t) \\ C(t) \end{cases},$$

$$\mathcal{Z}_0(t) = \begin{cases} S_0 \\ E_0 \\ I_0 \\ Q_0 \\ R_0 \\ D_0 \\ C_0 \end{cases}, \quad \Upsilon(t,\mathcal{Z}(t)) = \begin{cases} \mathbb{G}_1(S,E,I,Q,R,D,C,t) \\ \mathbb{G}_2(S,E,I,Q,R,D,C,t) \\ \mathbb{G}_3(S,E,I,Q,R,D,C,t) \\ \mathbb{G}_4(S,E,I,Q,R,D,C,t) \\ \mathbb{G}_5(S,E,I,Q,R,D,C,t) \\ \mathbb{G}_6(S,E,I,Q,R,D,C,t), \\ \mathbb{G}_7(S,E,I,Q,R,D,C,t). \end{cases} \tag{8}$$

Because of (6) and (7), we define the two operators $\mathbb{F}, \mathbb{G}$ from (7) as

$$\begin{aligned} \mathbb{F}(\mathcal{Z}) = &\quad \mathcal{Z}_0(t) + \tfrac{(1-s)}{\mathbb{ABC}(s)}[\Upsilon(t,\mathcal{Z}(t))], \\ \mathbb{G}(\mathcal{Z}) = &\quad \tfrac{s}{\mathbb{ABC}(s)\Gamma(s)}\int_0^t (t-y)^{s-1}\Upsilon(y,\mathcal{Z}(y))dy. \end{aligned} \tag{9}$$

Let us expressing some growth cognition and Lipschitzian assumption for existence and uniqueness as:

(A1) There exists constants $\mathbb{B}_\Upsilon, \mathbb{E}_\Upsilon$, such that

$|\Upsilon(t,\mathcal{Z}(t))| \leqslant \mathbb{B}_\Upsilon|\mathcal{Z}| + \mathbb{E}_\Upsilon$.

(A2) There exists constants $\mathbb{L}_\Upsilon > 0$ such that for each $\mathcal{Z}, \ \overline{\mathcal{Z}} \in \mathbf{Z}$ such that

$|\Upsilon(t,\mathcal{Z}) - \Upsilon(t,\overline{\mathcal{Z}})| \leqslant \mathbb{L}_\Upsilon[|\mathcal{Z}| - \overline{\mathcal{Z}}|]$;

Theorem 3.1. *Applying hypothesis* $(A1, A2)$*, the Integral* Eq. (7) *has at least one solution which consequently means that the considered system* (2) *has the same number of solution if* $\frac{(1-s)}{\mathbb{ABC}(s)}\mathbb{L}_\Upsilon < 1$.

Proof. We prove the theorem in two step as bellow:

Step I: Let $\overline{\mathcal{Z}} \in \mathbf{A}$, where $\mathbf{A} = \{\mathcal{Z} \in \mathbf{Z} : \ \|\mathcal{Z}\| \leqslant \sigma, \sigma > 0\}$ is closed convex set. Then using the definition of $\mathbb{F}$ in (9), one has

$$\begin{aligned} \|\mathbb{F}(\mathcal{Z}) - \mathbb{F}(\overline{\mathcal{Z}})\| = &\quad \tfrac{(1-s)}{\mathbb{ABC}(s)}\max_{t\in[0,\tau]}\left|\Upsilon(t,\mathcal{Z}(t)) - \Upsilon(t,\overline{\mathcal{Z}}(t))\right| \\ \leqslant &\quad \tfrac{(1-s)}{\mathbb{ABC}(s)}\mathbb{L}_\Upsilon\|\mathcal{Z} - \overline{\mathcal{Z}}\|. \end{aligned} \tag{10}$$

Hence $\mathbb{F}$ is contraction.

Step-II: To show that $\mathbb{G}$ is relatively compact, we show that $\mathbb{G}$ is bounded, and equi-continuous. Clearly $\mathbb{G}$ is continuous as Υ is continuous and also for any $\mathcal{Z} \in \mathbf{A}$, we have

$$\begin{aligned} \|\mathbb{G}(\mathcal{Z})\| = &\quad \max_{t\in[0,\tau]}\left\| \tfrac{s}{\mathbb{ABC}(s)\Gamma(s)}\int_0^t (t-y)^{s-1}\Upsilon(y,\mathcal{Z}(y))dy\right| \\ \leqslant &\quad \tfrac{s}{\mathbb{ABC}(s)\Gamma(s)}\int_0^\tau (\tau-y)^{s-1}|\Upsilon(y,\mathcal{Z}(y))|dy \\ \leqslant &\quad \tfrac{\tau^s}{\mathbb{ABC}(s)\Gamma(s)}[\mathbb{B}_\Upsilon\sigma + \mathbb{E}_\Upsilon]. \end{aligned} \tag{11}$$

Hence (11) implies that $\mathbb{G}$ is bounded. Further for equi-continuity let $t_1 > t_2 \in [0,\tau]$, we have

$$\begin{aligned} &|\mathbb{G}\Big(\mathcal{Z}(t_2) - \mathbb{G}\Big(\mathcal{Z}(t_1)| = \tfrac{s}{\mathbb{ABC}(s)\Gamma(s)}\Big|\int_0^{t_2} \\ &(t_2-y)^{s-1}\Upsilon(y,\mathcal{Z}(y))dy - \int_0^{t_1}(t_1-y)^{s-1}\Upsilon(y,\mathcal{Z}(y))dy| \\ &\quad \leqslant \frac{[\mathbb{B}_\Upsilon\sigma + \mathbb{E}_\Upsilon]}{\mathbb{ABC}(s)\Gamma(s)}\left[t_2^s - t_1^s\right]. \end{aligned} \tag{12}$$

Right side in (11) becomes zero at $t_2 \to t_1$. Since $\mathbb{G}$ is continues and so

$|\mathbb{G}(\mathcal{Z}(t_2) - \mathbb{G}(\mathcal{Z}(t_1)| \to 0, \ \text{as} \ t_2 \to t_1.$

Therefore we have as $\mathbb{G}$ is bounded operator and continuous so one has

$\|\mathbb{G}(\mathcal{Z}(t_2) - \mathbb{G}(\mathcal{Z}(t_1)\| \to 0, \ \text{as} \ t_2 \to t_1.$

So $\mathbb{G}$ is uniformly continuous and bounded. Thus by Arzelá-Ascoli theorem $\mathbb{G}$ is relatively compact and so completely continuous. Thus by Theorem 3.1, the integral Eq. (7) has at least one solution and consequently the system under consideration has at least one solution. □

For uniqueness we give the next result.

Theorem 3.2. *Under assumption* $(A2)$*, the integral Eq. (7) has unique solution which yields that the system under consideration (2) has unique result if* $\left[\frac{(1-s)\mathbb{L}_\Upsilon}{\mathbb{ABC}(s)} + \frac{\tau^s\mathbb{L}_\Upsilon}{\mathbb{ABC}(s)\Gamma(s)}\right] < 1$.

Proof. Let the operator $\mathbf{T}:\mathbf{Z}\to\mathbf{Z}$ defined by

$$\begin{aligned}\mathbf{T}\mathcal{Z}(t) = \quad & \mathcal{Z}_0(t) + [\Upsilon(t,\mathcal{Z}(t)) - \Upsilon_0(t)]\tfrac{(1-s)}{\mathbb{ABC}(s)} \\ & + \tfrac{s}{\mathbb{ABC}(s)\Gamma(s)}\int_0^t (t-y)^{s-1}\Upsilon(y,\mathcal{Z}(y))dy, \ \ t\in[0,\tau].\end{aligned} \tag{13}$$

Let $\mathcal{Z},\overline{\mathcal{Z}}\in\mathbf{Z}$, then one can take

$$\begin{aligned}|\mathbf{T}\mathcal{Z} - \mathbf{T}\overline{\mathcal{Z}}\| \leqslant & \tfrac{(1-s)}{\mathbb{ABC}(s)}\max_{t\in[0,\tau]}\left|\Upsilon(t,\mathcal{Z}(t)) - \Upsilon\left(t,\overline{\mathcal{Z}}(t)\right)\right| \\ & + \tfrac{s}{\mathbb{ABC}(s)\Gamma(s)}\max_{t\in[0,\tau]}\left|\int_0^t (t-y)^{s-1}\right. \\ \Upsilon(y,\mathcal{Z}(y))dy - & \left.\int_0^t (t-y)^{s-1}\Omega\left(y,\overline{\mathcal{Z}}(y)\right)dy\right| \\ \leqslant \Theta\|\mathcal{Z} - \overline{\mathcal{Z}}\|, &\end{aligned} \tag{14}$$

where

$$\Theta = \left[\frac{(1-s)\mathbb{L}_\Upsilon}{\mathbb{ABC}(s)} + \frac{\tau^s\mathbb{L}_\Upsilon}{\mathbb{ABC}(s)\Gamma(s)}\right]. \tag{15}$$

Thus $\mathbf{T}$ is contraction from (14). Therefore the integral Eq. (7) has unique solution. Hence our system (2) has unique solution. □

3.1. Numerical analysis

In this part of the paper, we are giving approximate solutions of fractional order model (2) under the $\mathbb{ABC}$ derivative by fractional Adams-Bashforth method. Then the numerical simulations are got via the suggested scheme. To this aim, we employ the fractional AB method [53] to establish a numerical procedure for the simulation of our considered model (2). To produce a numerical scheme, we go ahead with the model (5) can be written for simplicity as:

$$\begin{cases} {}^{\mathbb{ABC}}\mathcal{D}_t^s(S(t)) = \mathbb{G}_1(S(t),t) = \mu N_T + \tau C(t) - \alpha S(t) \\ -\beta\frac{S(T)I(t)}{N_T} - \mu S(t), \\ {}^{\mathbb{ABC}}\mathcal{D}_t^s(E(t)) = \mathbb{G}_2(E(t),t) = -\gamma E(t) + \beta\frac{S(T)I(t)}{N_T} - \mu E(t), \\ {}^{\mathbb{ABC}}\mathcal{D}_t^s(I(t)) = \mathbb{G}_3(I(t),t) = \gamma E(t) - \delta I(t) - \mu S(t) - \mu I(t), \\ {}^{\mathbb{ABC}}\mathcal{D}_t^s(Q(t)) = \mathbb{G}_4(Q(t),t) = \delta I(t) - \lambda(t)Q(t) \\ -k(t)Q(t) - \mu Q(t), \\ {}^{\mathbb{ABC}}\mathcal{D}_t^s(R(t)) = \mathbb{G}_5(R(t),t) = \lambda(t)Q(t) - k(t)Q(t) - \mu R(t), \\ {}^{\mathbb{ABC}}\mathcal{D}_t^s(D(t)) = \mathbb{G}_6(D(t),t) = k(t)Q(t), \\ {}^{\mathbb{ABC}}\mathcal{D}_t^s(D(t)) = \mathbb{G}_7(D(t),t) = \alpha S(t) - \mu C(t) - \tau C(t), \\ S(0) = S_0,\ E(0) = E_0,\ I(0) = I_0,\ Q(0) = Q_0\ R(0) \\ = R_0,\ D(0) = D_0,\ C(0) = C_0. \end{cases} \tag{16}$$

Taking integration of the first equation of (16) in $\mathbb{ABC}$ sense, we get

$$S(t) - S(0) = \frac{(1-s)}{\mathbb{ABC}(s)}[\mathbb{G}_1(S(t),t)] + \frac{s}{\mathbb{ABC}(s)\Gamma(s)} \times \int_0^t (t-y)^{s-1}\mathbb{G}_1(S(y),y)dy.$$

Set $t = t_{n+1}$ for $n = 0,1,2\cdots$, it follows that

$$\begin{aligned} S(t_{n+1}) - S(0) &= \tfrac{(1-s)}{\mathbb{ABC}(s)}[\mathbb{G}_1(S(t_n),t_n)] \\ + \tfrac{s}{\mathbb{ABC}(s)\Gamma(s)} & \int_0^{t_{n+1}}(t_{n+1}-y)^{s-1}\mathbb{G}_1(S(y),y)dy, \\ &= \tfrac{(1-s)}{\mathbb{ABC}(s)}[\mathbb{G}_1(S(t_n),t_n)] \\ + \tfrac{s}{\mathbb{ABC}(s)\Gamma(s)} & \sum_{p=0}^{n}\int_p^{t_{p+1}}(t_{n+1}-y)^{s-1}\mathbb{G}_1(S(y),y)dy. \end{aligned}$$

Now, we approximate the function $\mathbb{G}_1(S(t),t)$ on the interval $[t_p, t_{p+1}]$ through the interpolation polynomial as follows

$$\mathbb{G}_1(S(t),t) \cong \frac{\mathbb{G}_1(S(t_p),t_p)}{\Delta}(t-t_{p-1}) + \frac{\mathbb{G}_1(S(t_{p-1}),t_{p-1})}{\Delta}(t-t_p)$$

which implies that

$$\begin{aligned} S(t_{n+1}) = & S(0) + \tfrac{(1-s)}{\mathbb{ABC}(s)}[\mathbb{G}_1(S(t_n),t_n)] \\ & + \tfrac{s}{\mathbb{ABC}(s)\Gamma(s)}\sum_{p=0}^{n}\left(\tfrac{\mathbb{G}_1(S(t_p),t_p)}{\Delta}\int_p^{t_{p+1}}(t-t_{p-1})(t_{n+1}-t)^{s-1}dt\right. \\ & \left.-\tfrac{\mathbb{F}_1(S(t_{p-1}),t_{p-1})}{\Delta}\int_p^{t_{p+1}}(t-t_p)(t_{n+1}-t)^{p-1}dt\right) \\ = & S(0) + \tfrac{(1-s)}{\mathbb{ABC}(s)}[\mathbb{G}_1(S(t_n),t_n)] \\ & + \tfrac{s}{\mathbb{ABC}(s)\Gamma(s)}\sum_{p=0}^{n}\left(\tfrac{\mathbb{F}_1(S(t_p),t_p)}{\Delta}I_{p-1,s} - \tfrac{\mathbb{G}_1(S(t_{p-1}),t_{p-1})}{\Delta}I_{p,s}\right). \end{aligned} \tag{17}$$

Now we have to calculate the integrals $I_{p-1,s}$ and $I_{p,s}$ as

$$\begin{aligned} I_{p-1,s} = & \int_p^{t_{p+1}}(t-t_{p-1})(t_{n+1}-t)^{s-1}dt \\ = & -\tfrac{1}{s}\left[(t_{p+1}-t_{p-1})(t_{n+1}-t_{p+1})^s - (t_p-t_{p-1})(t_{n+1}-t_p)^s\right] \\ & -\tfrac{1}{s(s-1)}\left[(t_{n+1}-t_{p+1})^{s+1} - (t_{n+1}-t_p)^{s+1}\right], \end{aligned}$$

and

$$\begin{aligned} I_{p,s} = & \int_p^{t_{p+1}}(t-t_p)(t_{n+1}-t)^{s-1}dt \\ = & -\tfrac{1}{s}\left[(t_{p+1}-t_p)(t_{n+1}-t_{p+1})^r\right] \\ & -\tfrac{1}{s(s-1)}\left[(t_{n+1}-t_{p+1})^{s+1} - (t_{n+1}-t_p)^{s+1}\right], \end{aligned}$$

put $t_p = p\Delta$, we get

$$\begin{aligned} I_{p-1,s} & = -\tfrac{\Delta^{s+1}}{s}[(p+1-(p-1))(n+1-(p+1))^s - (p-(p-1))(n+1-p)^s] \\ & \quad -\tfrac{\Delta^{s+1}}{s(s-1)}\left[(n+1-(p+1))^{s+1} - (n+1-p)^{s+1}\right], \\ & = \tfrac{\Delta^{s+1}}{s(s-1)}\left[-2(s+1)(n-p)^s + (s+1)(n+1-q)^s - (n-q)^{s+1} + (n+1-q)^{s+1}\right], \quad (18) \\ & = \tfrac{\Delta^{s+1}}{s(s-1)}[(n-p)^s(-2(s+1)-(n-p)) + (n+1-p)^s(s+1+n+1-p)], \\ & = \tfrac{\Delta^{s+1}}{s(s-1)}[(n+1-p)^s(n-q+2+s) - (n-p)^s(n-p+2+2s)], \end{aligned}$$

and

$$
\begin{aligned}
I_{p,s} &= -\tfrac{\Delta^{s+1}}{s}\left[(p+1-p)(n+1-(p+1))^{s}\right] \\
&\quad -\tfrac{\Delta^{s+1}}{s(s-1)}\left[(n+1-(p+1))^{s+1}-(n+1-p)^{s+1}\right], \\
&= \tfrac{\Delta^{s+1}}{s(s-1)}\left[-(s+1)(n-p)^{s}-(n-p)^{s+1}+(n+1-p)^{s+1}\right], \\
&= \tfrac{\Delta^{s+1}}{s(s-1)}\left[(n-p)^{s}(-(p+1)-(n-p))+(n+1-p)^{s+1}\right], \\
&= \tfrac{\Delta^{s+1}}{s(s-1)}\left[(n+1-p)^{s+1}-(n-p)^{s}(n-p+1+s)\right],
\end{aligned} \tag{19}
$$

substituting (18) and (19) in (17), we have as follows

$$
\begin{aligned}
S(t_{n+1}) = {} & S(0)+\tfrac{(1-s)}{\mathbb{ABC}(s)}[\mathbb{G}_1(S(t_n),t_n)] \\
& +\tfrac{s}{\mathbb{ABC}(s)}\sum_{p=0}^{n}\Bigg(\tfrac{\mathbb{G}_1(S(t_p),t_p)}{\Gamma(s+2)}\Delta^{s}[(n+1-p)^{s}(n-p+2+s) \\
& -(n-p)^{s}(n-p+2+2s)] \\
& -\tfrac{\mathbb{G}_1(S(t_{p-1}),t_{p-1})}{\Gamma(s+2)}\Delta^{s}\Big[(n+1-p)^{s+1} \\
& -(n-p)^{s}(n-p+1+s)\Big]\Bigg).
\end{aligned}
$$

Similarly for the remaining six equations of (16) we can write the iterative method as

$$
\begin{aligned}
E(t_{n+1}) = {} & E(0)+\tfrac{(1-s)}{\mathbb{ABC}(s)}[\mathbb{G}_2(E(t_n),t_n)] \\
& +\tfrac{s}{\mathbb{ABC}(s)}\sum_{p=0}^{n}\Bigg(\tfrac{\mathbb{G}_2(E(t_p),t_p)}{\Gamma(s+2)}\Delta^{s}[(n+1-p)^{s}(n-p+2+s) \\
& -(n-p)^{s}(n-p+2+2s)] \\
& -\tfrac{\mathbb{G}_2(E(t_{p-1}),t_{p-1})}{\Gamma(s+2)}\Delta^{s}\Big[(n+1-p)^{s+1} \\
& -(n-p)^{s}(n-p+1+s)\Big]\Bigg).
\end{aligned}
$$

$$
\begin{aligned}
I(t_{n+1}) = {} & I(0)+\tfrac{(1-s)}{\mathbb{ABC}(s)}[\mathbb{G}_3(I(t_n),t_n)] \\
& +\tfrac{s}{\mathbb{ABC}(s)}\sum_{p=0}^{n}\Bigg(\tfrac{\mathbb{G}_3(I(t_p),t_p)}{\Gamma(s+2)}\Delta^{s}[(n+1-p)^{s}(n-q+2+s) \\
& -(n-p)^{s}(n-p+2+2s)] \\
& -\tfrac{\mathbb{G}_3(I(t_{p-1}),t_{p-1})}{\Gamma(s+2)}\Delta^{s}\Big[(n+1-p)^{s+1} \\
& -(n-p)^{s}(n-p+1+s)\Big]\Bigg).
\end{aligned}
$$

$$
\begin{aligned}
Q(t_{n+1}) = {} & Q(0)+\tfrac{(1-s)}{\mathbb{ABC}(s)}[\mathbb{G}_4(Q(t_n),t_n)] \\
& +\tfrac{s}{\mathbb{ABC}(s)}\sum_{p=0}^{n}\Bigg(\tfrac{\mathbb{G}_4(Q(t_p),t_p)}{\Gamma(s+2)}\Delta^{s}[(n+1-p)^{s}(n-q+2+s) \\
& -(n-p)^{s}(n-p+2+2s)] \\
& -\tfrac{\mathbb{G}_4(Q(t_{p-1}),t_{p-1})}{\Gamma(s+2)}\Delta^{s}\Big[(n+1-p)^{s+1} \\
& -(n-p)^{s}(n-p+1+s)\Big]\Bigg).
\end{aligned}
$$

$$
\begin{aligned}
R(t_{n+1}) &= R(0)+\tfrac{(1-s)}{\mathbb{ABC}(s)}[\mathbb{G}_5(R(t_n),t_n)] \\
& +\tfrac{s}{\mathbb{ABC}(s)}\sum_{p=0}^{n}\Bigg(\tfrac{\mathbb{G}_5(Q(t_p),t_p)}{\Gamma(s+2)}\Delta^{s}[(n+1-p)^{s}(n-q+2+s) \\
& -(n-p)^{s}(n-p+2+2s)] \\
& -\tfrac{\mathbb{G}_5(Q(t_{p-1}),t_{p-1})}{\Gamma(s+2)}\Delta^{s}\Big[(n+1-p)^{s+1} \\
& -(n-p)^{f}s(n-p+1+s)\Big]\Bigg).
\end{aligned}
$$

$$
\begin{aligned}
D(t_{n+1}) = {} & D(0)+\tfrac{(1-s)}{\mathbb{ABC}(s)}[\mathbb{G}_6(D(t_n),t_n)] \\
& +\tfrac{s}{\mathbb{ABC}(s)}\sum_{p=0}^{n}\Bigg(\tfrac{\mathbb{G}_6(D(t_p),t_p)}{\Gamma(s+2)}\Delta^{s}[(n+1-p)^{s}(n-q+2+s) \\
& -(n-p)^{s}(n-p+2+2s)] \\
& -\tfrac{\mathbb{G}_6(D(t_{p-1}),t_{p-1})}{\Gamma(s+2)}\Delta^{s}\Big[(n+1-p)^{s+1}-(n-p)^{s}(n-p+1+s)\Big]\Bigg).
\end{aligned}
$$

$$
\begin{aligned}
D(t_{n+1}) &= D(0)+\tfrac{(1-s)}{\mathbb{ABC}(s)}[\mathbb{G}_7(I(t_n),t_n)] \\
& \quad +\tfrac{s}{\mathbb{ABC}(s)}\sum_{p=0}^{n}\Bigg(\tfrac{\mathbb{G}_7(C(t_p),t_p)}{\Gamma(s+2)}\Delta^{s}[(n+1-p)^{s}(n-q+2+s) \\
& \quad -(n-p)^{s}(n-p+2+2s)]-\tfrac{\mathbb{G}_7(C(t_{p-1}),t_{p-1})}{\Gamma(s+2)}\Delta^{s}\Big[(n+1-p)^{s+1} \\
& \quad -(n-p)^{s}\Big(n-p+1+s\Big)\Big]\Bigg).
\end{aligned}
$$

3.2. Numerical simulations and discussion

Here we simulate our non-linear dynamical system data taken from [54]. By taking $\mu_{Malaysia} = 0.0000004$ million and $\tau = \frac{1}{80}$ and the compartments are taken as $S_0 = 32.37$ millions, $E_0 = 32.358052$ millions, $I_0 = 0.006726$ millions, $Q_0 = 0.006726$ millions, $R_0 = 0.005222$ millions, $D_0 = 0.000108$ millions, $C_0 = 0.088313$ millions and for all other parameters are given in Table 2 as.

In Figs. 1–7, we have presented by simulation the spreading of COVID-19 in Malaysia during the last eighty days that is from 21^{st} of February, 2020 to 11^{th} of May 2020 by taking real data[58]. From the plot we see that as the susceptible population decreasing at very slow rate on different fractional order. Smaller the order faster the decay and vice versa. Also the exposed class raises with slow rate if the fractional order is small as compared to greater order. As a results the infected class, the quarantined class, the confined susceptible class, recovered and death cases have increased but with slow rate. The concerned rate is faster if the fractional order is greater and vice versa. Also the results inclines to the classical solution of the fractional order tends to integer 1.

Next we simulate the considered model (2) for Pakistan. According to [55], for Pakistan we take $\mu_{Pakistan} = 0.00880$ per million and $\tau = \frac{1}{80}$ and the compartments are taken as $S_0 = 220.892$ millions, $E_0 = 220.81857$ millions, $I_0 = 0.032081$ millions, $Q_0 = 0.032081$ millions, $R_0 = 0.008555$ millions, $D_0 = 0.000706$ millions, $C_0 = 80.7777$ (assumed) and for all other parameters as given in Table 3 as.

In Figs. 8–14 we have presented the aforesaid infection in Pakistan during the last eighty days. We seen that the decay in susceptible class in Pakistan is slightly rapid as compared to Malaysia. Because the protection rate and the sensibility about the disease in peoples of Pakistan are less than those of the Malaysian peoples. The increase in exposed class, infection class, quarantined class, death class, recovered class are raised with rapid speed. The concerned growth rate is faster on greater fractional order as compared to small fractional order. In Pakistan the population of Confined susceptible people has deceased in the last eighty days with different fractional order. This decay is faster at small fractional order but slow at

Table 2 Numerical values of parameters given in model (1), for the spreading of COVID-19 in Malaysia.

Scenario	α	β	γ	δ	λ_0	λ_1	k_0	k_1
Malaysia	0.015	0.000024	0.200	0.0610	0.75	0.0000004	0.017	0.051

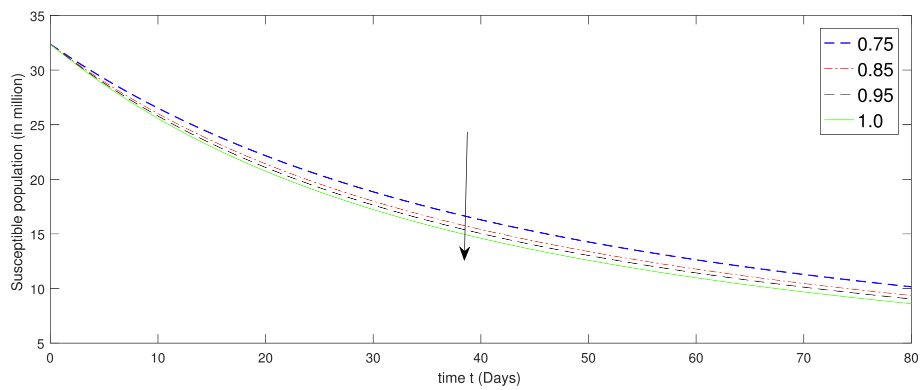

Fig. 1 Dynamical behavior of susceptible class (population) for the considered model (2) corresponding to different fraction order derivative.

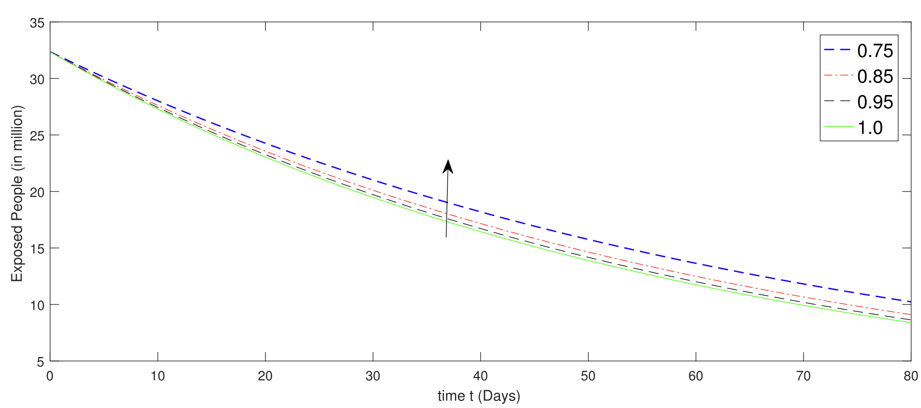

Fig. 2 Dynamical behavior of exposed class (population) for the considered model (2) corresponding to different fraction order derivative.

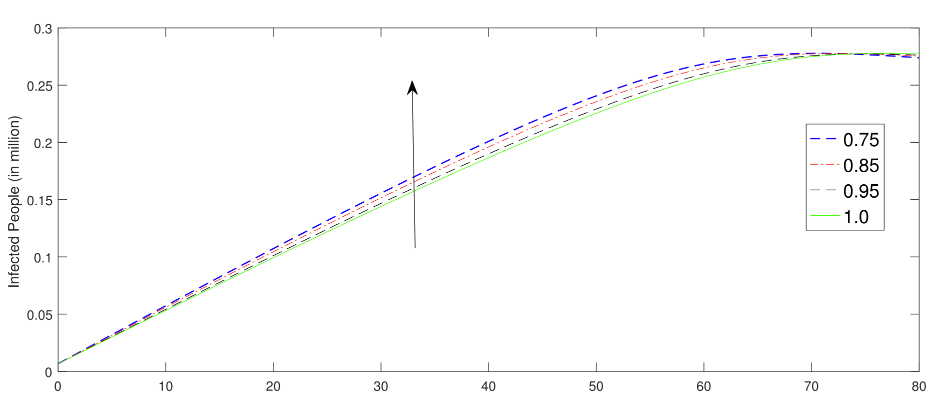

Fig. 3 Dynamical behavior of infected class (population) for the considered model (2) corresponding to different fraction order derivative.

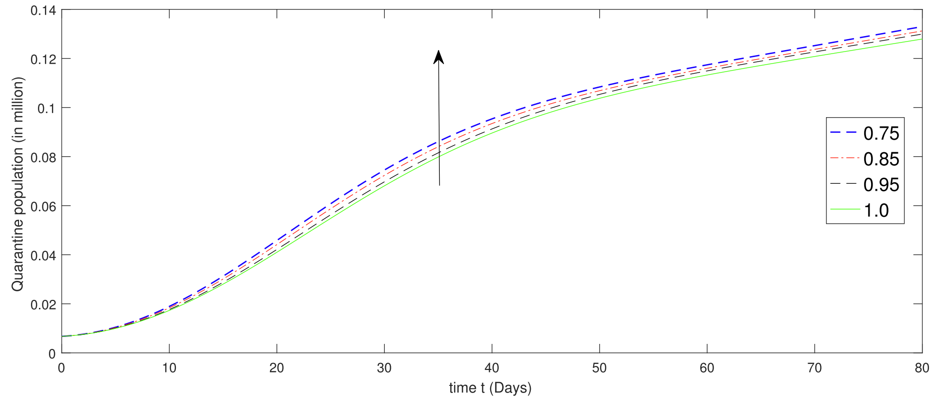

Fig. 4 Dynamical behavior of quarantined class (population) for the considered model (2) corresponding to different fraction order derivative.

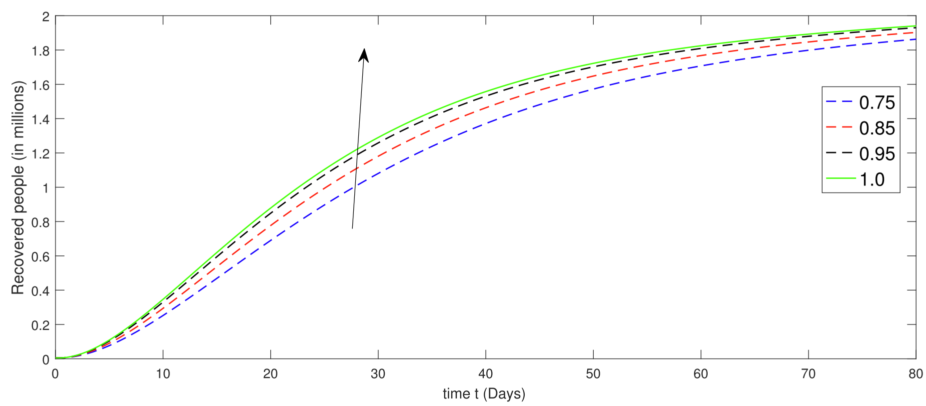

Fig. 5 Dynamical behavior of recovered class (population) for the considered model (2) corresponding to different fraction order derivative.

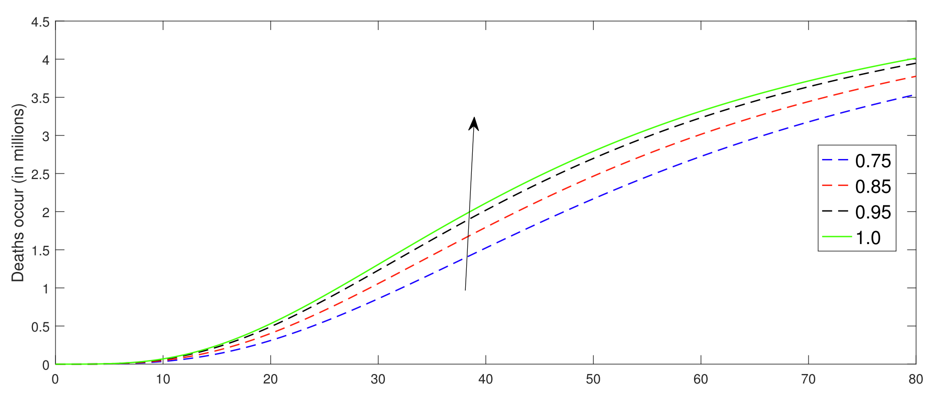

Fig. 6 Dynamical behavior of death class (population) for the considered model (2) corresponding to different fraction order derivative.

greater fractional order. The decay in class of confined class has also caused the increase in infection.

If we compare the above plots for the last eighty days between the two countries, we see that infection has been spread in Pakistan with high speed as compared to Malaysia. There are many causes of this, like the protection rate in Pakistan is less as compared to Malaysia. Also, the people in Pakistan has not taken the issue serious and which caused the increase in infection. One of the big reason is that nearly 70 percent people in Pakistan are living under poor conditions and only 30 percent people (assumed) are confined class which take care about the current infection. Another great cause of infection in Pakistan is huge population as compared to Malaysia. Therefore in last eighty days the concerned infection spread with rapid speed in Pakistan as compared to Malaysia.

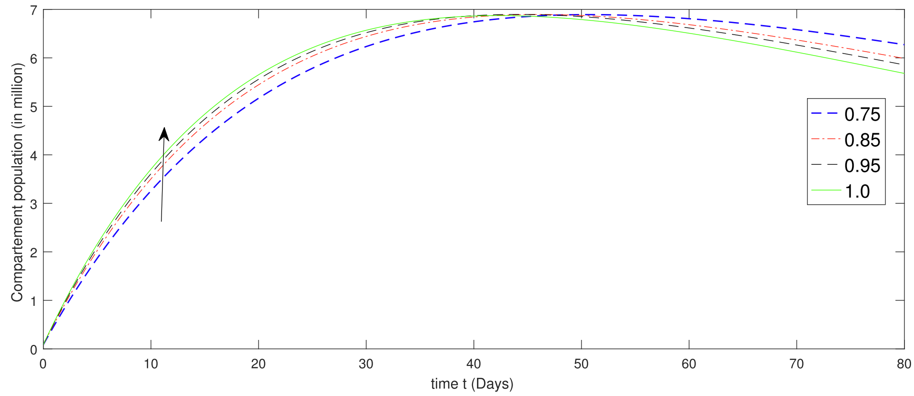

Fig. 7 Dynamical behavior of protected compartment class (population) for the considered model (2) corresponding to different fraction order derivative.

Table 3 Numerical values of parameters used in model (1), for the spreading of COVID-19 in Pakistan.

Scenario	α	β	γ	δ	λ_0	λ_1	k_0	k_1
Pakistan	0.000761	0.073	0.0009	0.0610	0.000791	0.027	0.020	0.0619

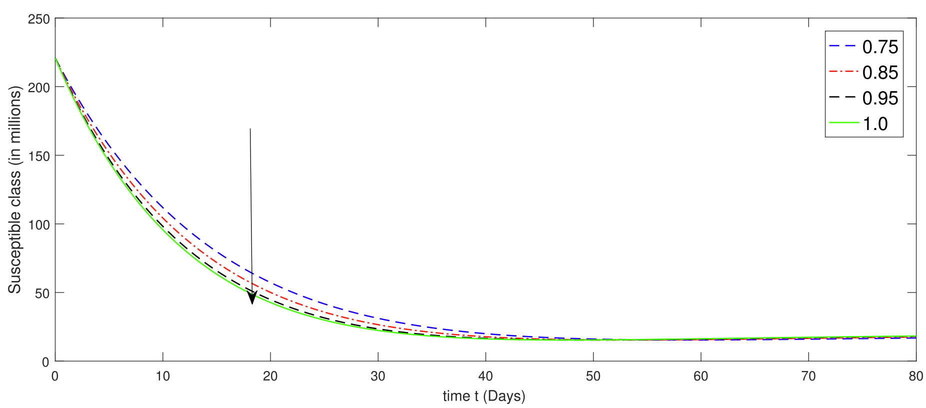

Fig. 8 Dynamical behavior of susceptible class (population) for the considered model (2) corresponding to different fraction order derivative.

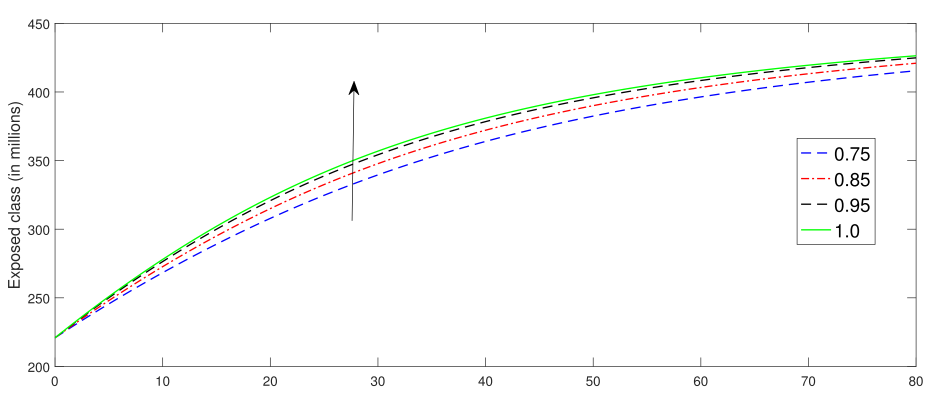

Fig. 9 Dynamical behavior of exposed class (population) for the considered model (2) corresponding to different fraction order derivative.

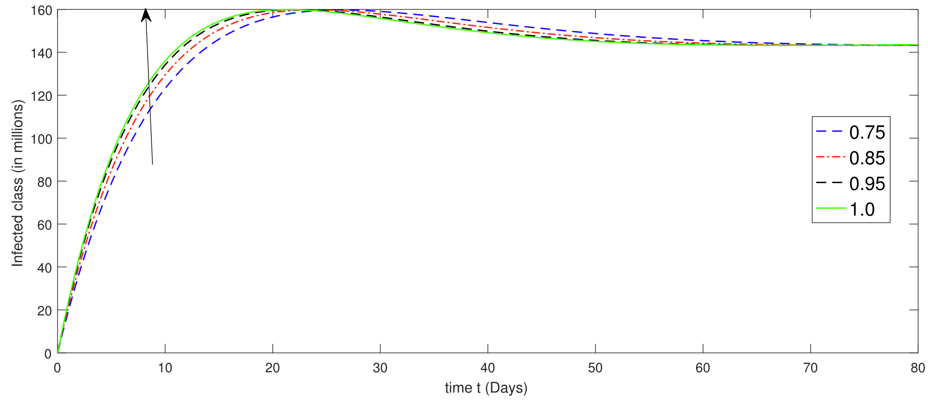

Fig. 10 Dynamical behavior of infected class (population) for the considered model (2) corresponding to different fraction order derivative.

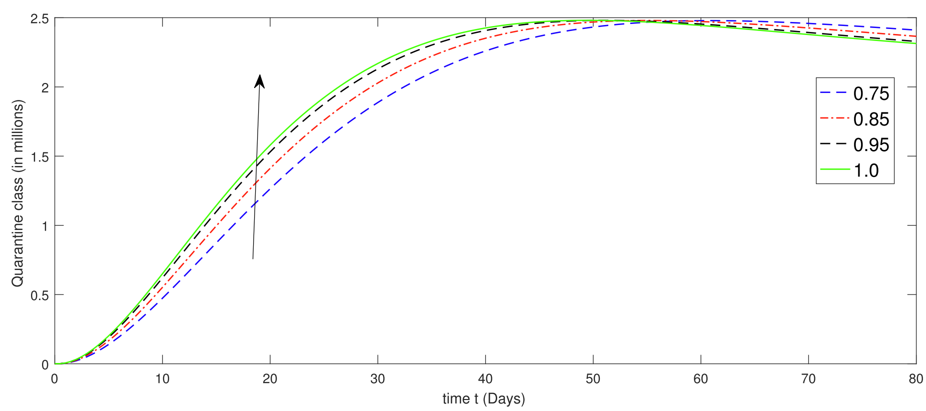

Fig. 11 Dynamical behavior of quarantined class (population) for the considered model (2) corresponding to different fraction order derivative.

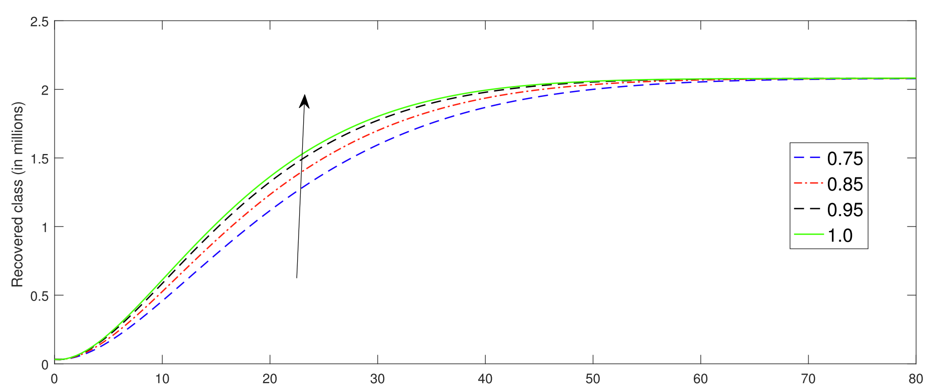

Fig. 12 Dynamical behavior of recovered class (population) for the considered model (2) corresponding to different fraction order derivative.

4. Conclusion

A modified type SEIR model under nonsingular fractional order derivative has investigated from theoretical and numerical aspects against real data of Malaysia and Pakistan. We have proved the existence and uniqueness of the model by fixed point approach. Further on using Adam-Bashforth method, we have established a numerical scheme to simulate the results by using Matlab. From the simulation we have observed that due to less protection rate, huge population, bad health conditions, poverty and the weak precautionary measures in last eighty days, the COVID-19 has increasingly spread in Pakistan as compared to Malaysia. Due to which more infected and fatality cases have occurred in Pakistan. On the other hand

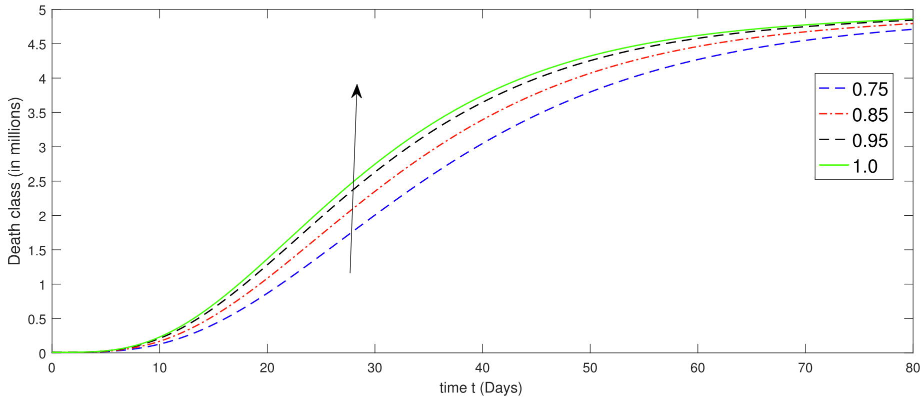

Fig. 13 Dynamical behavior of death class (population) for the considered model (2) corresponding to different fraction order derivative.

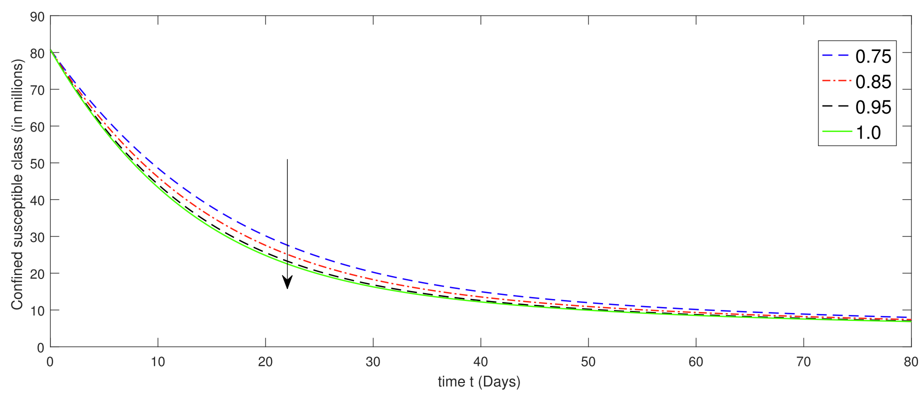

Fig. 14 Dynamical behavior of protected compartment class (population) for the considered model (2) corresponding to different fraction order derivative.

the condition of Malaysia is better due to better health condition, more protection rate, etc.

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Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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