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Research Article

Computation of solution to fractional order partial reaction diffusion equations

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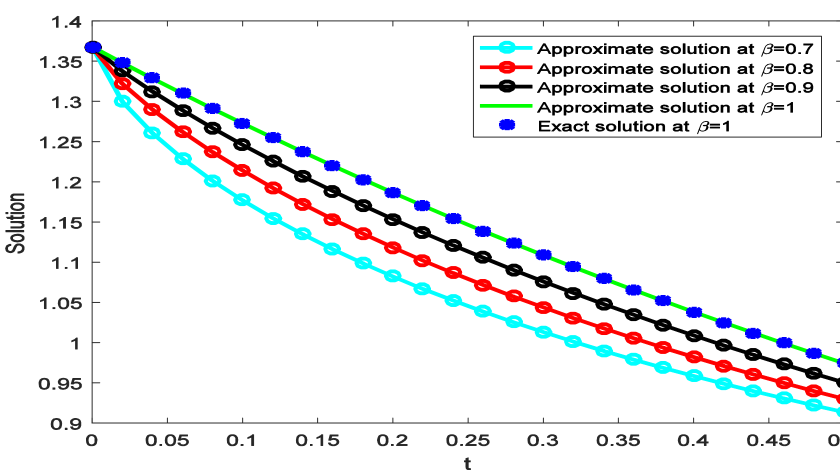
HIGHLIGHTS

- Applying the proposed novel method (PNM) to find the approximate solution of fractional order CRDE.
- The PNM to fractional order CRDE gives more realistic series solutions that converge very rapidly.
- PNM is very simple, effective and accurate as compared to other analytical techniques.

GRAPHICAL ABSTRACT



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ABSTRACT

In this article, the considered problem of Cauchy reaction diffusion equation of fractional order is solved by using integral transform of Laplace coupled with decomposition technique due to Adomian scheme. This combination led us to a hybrid method which has been properly used to handle nonlinear and linear problems. The considered problem is used in modeling spatial effects in engineering, biology and ecology. The fractional derivative is considered in Caputo sense. The results are obtained in series form corresponding to the proposed problem of fractional order. To present the analytical procedure of the proposed method, some test examples are provided. An approximate solution of a fractional order diffusion equation were obtained. This solution was rapidly convergent to the exact solution with less computational cost. For the computation purposes, we used MATLAB.

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Introduction

Indeed fractional calculus is an important field of applied mathematics in recent decade. Using fractional derivatives and fractional integrals to model real world phenomenons give better results than classical order. Some interesting applications can be traced in modeling several physical phenomenons, particularly, in the field of the damping visco-elasticity, electronic, signal processing, biology, genetic algorithms, robotic technology, telecommunication, traffic systems, chemistry, physics as well as economics and finance. Many researchers have devoted some important developments and contributions to the field of fractional calculus [1–8]. Due to large interesting usage, fractional calculus is considered as very important field of research for most of the researchers and scientists. In the field of fractional calculus, the study of fractional order partial differential equations (FOPDEs) has particularly been focused by many researchers. In this concern, linear and non-linear FODEs have been solved via using various methods. For instance, analysis of modified Bernoulli sub-equation and non-linear time fractional Burgers equations has been presented in [9]. The numerical simulation to space fractional diffusion equations have been performed in [10,11]. The exact solutions of nonlinear biological population models of fractional order has been obtained in [12] by optimal homotopy method (OHAM). On using OHAM, the solution of Burgers- Huxley models [13] has been computed. Investigations of nonlinear FOPDEs via homotopy perturbation transform method was performed in [14]. In same line, the approximate solution to generalized Mittag -Leffler law via exponential decay has been discussed in [15]. Moreover, various applications of derivatives and integral of arbitrary order have been discussed in [16]. For the development of this field, In [17,18], some researchers gave the numerical schemes and stability for two classes of FOPDEs.

On other hand, obtaining the exact as well as an approximate solutions of FOPDEs is the main interest of many researchers. In this concern, in 2001, a proposed novel method (LADM) was applied, for the first time, by Khuri for the solution of ODEs. Thereafter, it has been successfully applied for the solution of many classical PDEs in engineering and natural sciences. LADM is the combination of two powerful methods that is decomposition and integral transform, (for detail see [19,20]). Many physical phenomena which have been modeled by PDEs and FOPDEs were solved by using LADM. For instance, the analytical solution of Whitham-Broer-Kaup equations has been computed in [21]. Further, the solution of linear and non-linear FOPDEs were successfully presented in[22]. Authors [23] have discussed the numerical solution of nonlinear fractional Volterra Fredholm integro-differential equations. In same line, system of fractional delay differential equations have been successfully described in [24]. Also, the solution of well known diffusion equation has been presented in [25] and for some applications of proposed method to non-linear FOPDEs, (we refer [26]).

In this article, we contribute to the field of approximate/ exact analytical solutions of applied problems which occur in engineering and many physical phenomena. In this concern, we extend LADM for the approximate solution of reaction–diffusion equation (RDE) of fractional order and its various cases. The RDE of fractional order [27–29] is provided as:

$$\frac{\partial^\beta z(\xi,t)}{\partial t^\beta} = c\frac{\partial^2 z(\xi,t)}{\partial \xi^2} + r(\xi,t)z(\xi,t), \quad (\xi,t)\in\Omega. \tag{1}$$

The problem (1) becomes classical RDE if $\beta = 1$. In the Eq. (1), the term $c(\xi,t)\frac{\partial^2 z(\xi,t)}{\partial \xi^2}$ denotes diffusion and $r(\xi,t)z(\xi,t)$ denotes the reaction, where $r(\xi,t)$ reaction parameter, $z(\xi,t)$ is the concentration and c is diffusion coefficient constant.

Moreover, we refer to recent papers devoted to the analytical and theoretical studies of the time-fractional diffusion equation [30–33].

Preliminaries

Here, in this section we provide background materials of basic definitions and some known results of the fractional calculus. Also some important preliminaries are recalled from the field of applied analysis.

Definition 2.1. [34] "Riemann–Liouville integral of fractional order" $\beta\in R^+$ for the function $h\in L([0,1],R)$ is given as:

$$I_0^\beta h(t) = \frac{1}{\Gamma(\beta)}\int_0^t (t-s)^{\beta-1}h(s)ds, \tag{2}$$

provided that integral exists (on right hand side).

Definition 2.2. [34] For the $p\in\mathbb{R}$, a function $f:\mathbb{R}\to\mathbb{R}^+$ is said to be in the space C_p if it can be written as $f(\xi)=\xi^q f_1(\xi)$with $q>p,\ f_1(\xi)\in C[0,\infty)$ such that $f(\xi)\in C_p^m$ if $f^{(m)}\in C_p$ for $m\in\mathbb{N}\cup\{0\}$.

Definition 2.3. [34] Caputo fractional derivative of a function $h\in C_{-1}^m$ with $m\in\mathbb{N}\cup\{0\}$ is provided as:

$$D_\xi^\beta h(\xi) = \begin{cases} I^{m-\beta}f^{(m)}, & m-1<\beta\leqslant m,\ m\in\mathbb{N}, \\ \frac{d^m}{d\xi^m}h(\xi), & \beta=m,\ m\in\mathbb{N}. \end{cases} \tag{3}$$

Definition 2.4. [34] The two parameter Mittag–Leffler function is provided as:

$$\mathrm{E}_{\alpha,\beta}(t) = \sum_{k=0}^{\infty}\frac{t^k}{\Gamma(k\alpha+\beta)}. \tag{4}$$

If $\alpha=\beta=1$ in (4), we obtain $\mathrm{E}_{1,1}(t)=e^t$ and $\mathrm{E}_{1,1}(-t)=e^{-t}$.

Definition 2.5. [35] Laplace transformation (LT) of the function $g(\xi),\ \xi>0$is provided as:

$$G(s) = L[g(\xi)] = \int_0^\infty e^{-s\xi}g(\xi)d\xi,$$

where s can be either real or complex.

Definition 2.6. [35] LT in terms of the convolution is defined as:

$$L[g_1\times g_2] = L[g_1]\times L[g_2],$$

where $g_1\times g_2$ is defined by (shows the convolution between g_1 and g_2)

$$(g_1\times g_2)\xi = \int_0^\infty g_1(t)g_2(\xi-t)d\xi.$$

The LT of Caputo derivatives is defined as:

$$L\left[D_\xi^\beta g(\xi)\right] = s^\beta G(s) - \sum_{k=0}^{n-1}s^{\beta-1-k}g^{(k)}(0),\ \ n-1<\beta<n.$$

Construction of the method

Here, in this section, we discuss how to establish LADM [21] to solve RDE of fractional order and its various cases.

The RDE with fractional order and its formulation by LADM are given as

$$\frac{\partial^\beta z(\xi,t)}{\partial t^\beta} = c\frac{\partial^2 z(\xi,t)}{\partial \xi^2} + r(\xi,t)z(\xi,t),\ (\xi,t)\in\Omega \tag{5}$$

with initial condition

$$z(\xi,0) = g(\xi).$$

Now we apply the LT on Eq. (5)

$$L\left[\frac{\partial^\beta z(\xi,t)}{\partial t^\beta}\right] = cL\left[\frac{\partial^2 z(\xi,t)}{\partial \xi^2}\right] + L[r(\xi,t)z(\xi,t)].$$

Using the differentiation properties of LT, we obtain

$$L[z(\xi,t)] = \frac{g(\xi)}{s} + \frac{1}{s^\beta}L[r(\xi,t)z(\xi,t)] + cL\frac{1}{s^\beta}\left[\frac{\partial^2 z(\xi,t)}{\partial \xi^2}\right]. \tag{6}$$

Consider the solutions $z(\xi,t)$ in the form as

$$z(\xi,t) = \sum_{j=0}^{\infty} z_j(\xi,t).$$

The nonlinear terms show that infinite series of the Adomian polynomials,

$$N_1(z(\xi,t)) = \sum_{j=0}^{\infty} A_j,$$

$$A_j = \frac{1}{j!}\left[\frac{d^j}{d\lambda^j}\left(N_1\sum_{i=0}^{\infty}\lambda^j z_i\right)\right].$$

Hence the Eq. (6) is

$$L\left[\sum_{j=0}^{\infty} z_{j+1}\right] = \frac{g(\xi)}{s} + \frac{1}{s^\beta}L\left[c\frac{\partial^2}{\partial\xi^2}\sum_{j=0}^{\infty} z_j(\xi,t) + r(\xi,t)\sum_{j=0}^{\infty} z_j(\xi,t)\right].$$

Applying the linearity of LT, we have

$$L[z_0(\xi,t)] = \frac{g(\xi)}{s},$$

$$L\left[\sum_{j=0}^{\infty} z_{j+1}\right] = \frac{1}{s^\beta}L\left[c\frac{\partial^2}{\partial\xi^2}\sum_{j=0}^{\infty} z_j + r\sum_{j=0}^{\infty} z_j\right],$$

where $r = r(\xi,t)$, for $j = 0,1,2,3,\ldots$.

By applying inverse LT, we can obtain $z_0, z_1, z_2, \ldots$.

Therefore, the series solution is given by

$$\tilde{z}(\xi,t) = z_0 + z_1 + z_2 + \ldots.$$

Test Problems

Here, in this section, we provide the easy and smooth convergence of LADM for the solutions of some test problems which are special cases of CRDE of fractional order.

Example 4.1. We study the LADM for a special case of FOPDEs (1) at positive t

$$\frac{\partial^\beta z(\xi,t)}{\partial t^\beta} = \frac{\partial^2 z(\xi,t)}{\partial \xi^2} - z(\xi,t),\ \beta\in(0,1], \tag{7}$$

with initial condition

$$z(\xi,0) = e^{-\xi} + \xi.$$

Now, we apply the LT of Eq. (7)

$$L\left[\frac{\partial^\beta z(\xi,t)}{\partial \xi^\beta}\right] = L\left[\frac{\partial^2 z(\xi,t)}{\partial \xi^2} - z(\xi,t)\right],$$

$$s^\beta z(\xi,t) - s^{\beta-1}z(\xi,0) = L\left[\frac{\partial^2 z(\xi,t)}{\partial \xi^2} - z(\xi,t)\right].$$

According to Laplace inverse transform, we have $z_0(\xi,t) = L^{-1}\left[\frac{z(\xi,0)}{s}\right], z_{j+1}(\xi,t) = L^{-1}\frac{1}{s^\beta}L\left[\frac{\partial^2 z_j(\xi,t)}{\partial\xi^2} - z_j(\xi,t)\right]$, for $j = 0,1,2,\ldots$.

Therefore, we obtain

$$z_0(\xi,t) = e^{-\xi} + \xi,$$

$$z_1(\xi,t) = -\frac{\xi t^\beta}{\Gamma(\beta+1)},$$

$$z_2(\xi,t) = \frac{\xi t^{2\beta}}{\Gamma(2\beta+1)},$$

$$z_3(\xi,t) = -\frac{\xi t^{3\beta}}{\Gamma(3\beta+1)},$$

$$z_4(\xi,t) = \frac{\xi t^{4\beta}}{\Gamma(4\beta+1)}.$$

Similarly, we can find $z_5, z_6, \ldots$.

Hence, the series solution becomes

$$\tilde{z}(\xi,t) = e^{-\xi} + \xi\left[1 - \frac{t^\beta}{\Gamma(\beta+1)} + \frac{t^{2\beta}}{\Gamma(2\beta+1)} - \frac{t^{3\beta}}{\Gamma(3\beta+1)} + \frac{t^{4\beta}}{\Gamma(4\beta+1)}\ldots\right], \tag{8}$$

$$\tilde{z}(\xi,t) = e^{-\xi} + \xi E_\beta(-t^\beta). \tag{9}$$

When $\beta = 1$, then Eq. (9) becomes the exact solution of RDE of integer order [27,28].

For accuracy and simplicity of the LADM, truncating the solution in (8) at level $n = 12$. Numerical results of Example 4.1 are shown in Tables 1, 2 which are also plotted in Figs. 1–3. The results in Table 2 and Fig. 1 (Green line shows approximate solution and blue dots line shows exact solution) provide the comparison of exact and LADM approximate solutions at $\beta = 1$. A surface graph of the solutions of Example 4.1 is plotted in Fig. 2, wherein for simple execution of the Matlab code, we have replaced $\tilde{z}(\xi,t)$ by $w(x,t)$. Each plot in the figures has the demonstration of physical behavior of the approximate solutions. Moreover, the absolute error are plotted in Fig. 3. It shows significance indication that the exact and approximate solutions are closed to each others.

Table 1
Solutions of Problem 4.1 by LADM for various value of the t at $\xi = 1$ and taking $\beta = 0.7, 0.8, 0.9$.

t	LADM($\beta = 0.7$)	LADM($\beta = 0.8$)	LADM($\beta = 0.9$)
0	1.36787944117	1.36787944117	1.36787944117
0.04	1.26063785322	1.29003540632	1.3122734338
0.08	1.20140342889	1.23708540691	1.2668713807
0.12	1.15498262073	1.19258355139	1.2260185458
0.16	1.11606160395	1.15352738033	1.18848488707
0.20	1.0823166425	1.11850470929	1.15364185989
0.24	1.05245054008	1.08667976807	1.12108982079
0.28	1.02564040204	1.05749488452	1.09054418331
0.32	1.00132074622	1.0305491701	1.06178776421
0.36	0.979080995722	1.00553943111	1.03464697699
0.40	0.958610800117	0.982227775522	1.0089784682
0.44	0.939668244664	0.960422291871	0.984660961807
0.48	0.922060129646	0.939964769682	0.961589956907

Table 2
Absolute error of LADM results of Problem 4.1 for various value of the t at $\xi = 1$ and taking $\beta = 1$.

t	$Exact solution(\beta = 1)$	$LADM solution(\beta = 1)$	$Error$
0	1.36787944117	1.36787944117	0
0.04	1.32866888032	1.32866888032	0
0.08	1.29099578756	1.29099578756	0
0.12	1.25479987789	1.25479987789	0
0.16	1.22002323014	1.22002323014	0
0.20	1.18661019425	1.18661019425	0
0.24	1.15450730224	1.15450730224	0
0.28	1.12366318263	1.12366318263	$1.04e-17$
0.32	1.09402847825	1.09402847825	$5.9e-17$
0.36	1.06555576724	1.06555576724	$2.67e-16$
0.40	1.03819948721	1.03819948721	$1.05e-15$
0.44	1.01191586225	1.01191586225	$3.61e-15$
0.48	0.986662832978	0.986662832978	$1.11e-14$

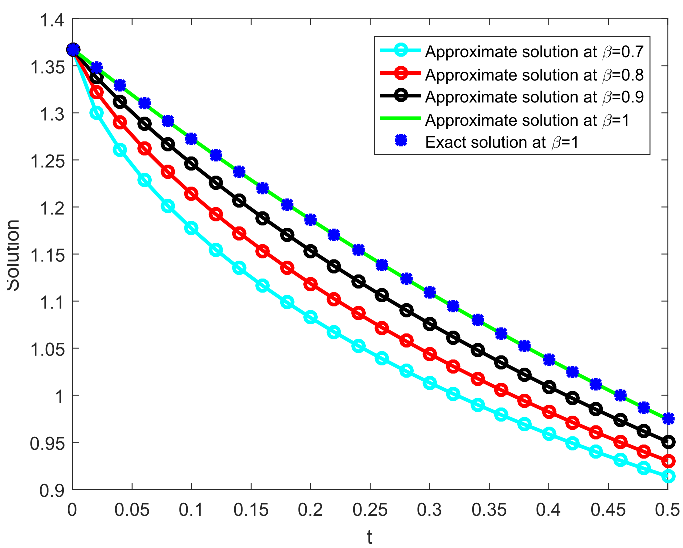

Fig. 1. Comparison of exact and LADM results of the Problem 4.1 at $\xi = 1$ for various values of t and β.

Example 4.2. We study the LADM for another special case at $t > 0$ of RDE (1),

$$\frac{\partial^{\beta} z(\xi,t)}{\partial t^{\beta}} = \frac{\partial^{2} z(\xi,t)}{\partial \xi^{2}} - (1+4\xi^{2}) z(\xi,t),\ \beta \in (0,1], \tag{10}$$

with initial condition

$$z(\xi,0) = e^{\xi^2}.$$

We apply LT method to Eq. (10) as

$$L\left[\frac{\partial^{\beta} z(\xi,t)}{\partial t^{\beta}}\right] = L\left[\frac{\partial^{2} z(\xi,t)}{\partial \xi^{2}} - (1+4\xi^{2}) z(\xi,t)\right],$$

$$s^{\beta} z(\xi,t) - s^{\beta-1} z(\xi,0) = L\left[\frac{\partial^{2} z(\xi,t)}{\partial \xi^{2}} - (1+4\xi^{2}) z(\xi,t)\right].$$

Therefore, according to inverse LT

$$z_0(\xi,0) = L^{-1}\left[\frac{z(\xi,0)}{s}\right],$$

$$z_{j+1}(\xi,t) = L^{-1}\frac{1}{s^{\beta}}\left[L\left[\frac{\partial^{2} z_j(\xi,t)}{\partial \xi^{2}} - (1+4\xi^{2}) z_j(\xi,t)\right]\right],$$

for $j = 0,1,2,\ldots$.
We compute

$$z_0(\xi,t) = e^{\xi^2},$$

$$z_1(\xi,t) = \frac{e^{\xi^2} t^{\beta}}{\Gamma(\beta+1)},$$

$$z_2(\xi,t) = \frac{e^{\xi^2} t^{2\beta}}{\Gamma(2\beta+1)},$$

$$z_3(\xi,t) = \frac{e^{\xi^2} t^{3\beta}}{\Gamma(3\beta+1)}.$$

Similarly, we can find $z_4, z_5, \ldots$.
Hence, the series solution becomes

$$\tilde{z}(\xi,t) = e^{\xi^2}\left[1 + \frac{t^{\beta}}{\Gamma(\beta+1)} + \frac{t^{2\beta}}{\Gamma(2\beta+1)} + \frac{t^{3\beta}}{\Gamma(3\beta+1)} + \ldots\right], \tag{11}$$

$$\tilde{z}(\xi,t) = e^{\xi^2} E_{\beta}(t^{\beta}). \tag{12}$$

When $\beta = 1$, then solution in Eq. (12) is transferred to

$$\tilde{z}(\xi,t) = e^{\xi^2 + t}, \tag{13}$$

which is the exact solution of the RDE of integer order that is obtained in [27,28].

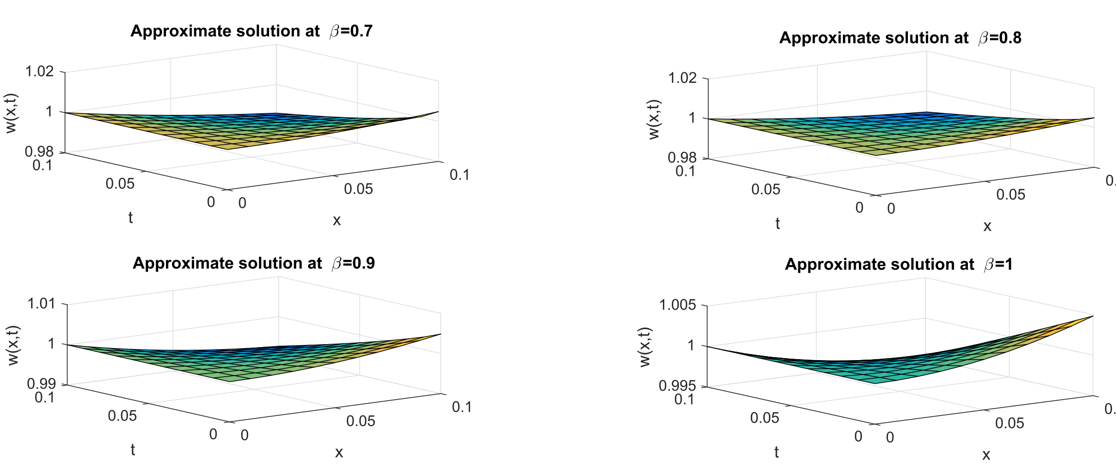

Fig. 2. LADM results of the Problem 4.1 for various values of $x(\xi)$, t and β.

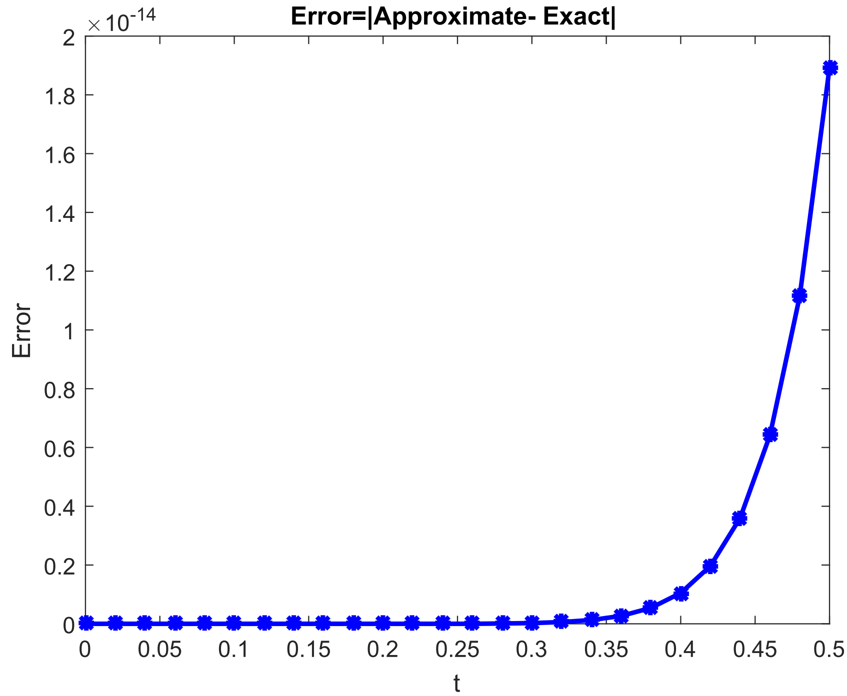

Fig. 3. Absolute error plot of LADM results of the Problem 4.1 for various values of t and $\beta = 1$.

For accuracy and simplicity of the LADM, truncating the solution in (11) at level $n = 12$. Numerical results of Example 4.2 are shown in Tables 3, 4 and have been plotted in the Figs. 4–6. The results in Table 4 and Fig. 4 (Green line shows approximate solution and blue dots line shows exact solution) provide the comparison of exact and LADM approximate solutions at $\beta = 1$. A surface graph of the solutions of Example 4.2 is plotted in Fig. 5, wherein for simple execution of the Matlab code, we have replaced $\tilde{z}(\xi, t)$ by $w(x, t)$. Each plot in the figures has the demonstration of physical behavior of the approximate solutions. Moreover, the absolute error are plotted in Fig. 6. They show significance indication that the exact and approximate solutions are very closed to each others.

Example 4.3. We study the LADM for another special case $t > 0$ of FOPDEs (1)

$$\frac{\partial^\beta z(\xi,t)}{\partial t^\beta} = \frac{\partial^2 z(\xi,t)}{\partial \xi^2} - (2 + 4\xi^2 - 2t)z(\xi,t),\ \beta \in (0,1], \tag{14}$$

Table 3
Results of Problem 4.2 by LADM corresponding to various value of t at $\xi = 1$ and taking $\beta = 0.7, 0.8, 0.9$.

t	$LADM(\beta = 0.7)$	$LADM(\beta = 0.8)$	$LADM(\beta = 0.9)$
0	2.71828182846	2.71828182846	2.71828182846
0.04	3.05824497161	2.95195691542	2.87931553947
0.08	3.29928547606	3.14087530059	3.02729013991
0.12	3.52498051186	3.32345744113	3.17545582751
0.16	3.74615701638	3.50557634111	3.3262496936
0.20	3.96731861322	3.68981262922	3.48085001233
0.24	4.19095246225	3.87766326559	3.64000821733
0.28	4.41867367697	4.07014825832	3.80428597497
0.32	4.65165413111	4.26804311495	3.97414838383
0.36	4.8908169499	4.47198611344	4.15000791035
0.40	5.13693654863	4.68253429259	4.33224777814
0.44	5.3906949783	4.90019541171	4.52123558949
0.48	5.65271595406	5.12544750429	4.71733184507

Table 4
Absolute error of LADM results of Problem 4.2 corresponding to various value of t at $\xi = 1$ and taking $\beta = 1$.

t	$Exact\,solution(\beta = 1)$	$LADMsolution(\beta = 1)$	$Error$
0	2.71828182846	2.71828182846	0
0.04	2.82921701435	2.82921701435	$6.94e-18$
0.08	2.94467955107	2.94467955107	$6.94e-18$
0.12	3.06485420329	3.06485420329	0
0.16	3.18993327612	3.18993327612	$6.94e-18$
0.20	3.32011692274	3.32011692274	$6.94e-18$
0.24	3.45561346476	3.45561346476	$1.39e-17$
0.28	3.59663972557	3.59663972557	$2.78e-17$
0.32	3.74342137726	3.74342137726	$1.67e-16$
0.36	3.8961933018	3.8961933018	$7.70e-16$
0.40	4.05519996684	4.05519996684	$3.03e-15$
0.44	4.220695817	4.220695817	$1.04e-14$
0.48	4.39294568092	4.39294568092	$3.24e-14$

with initial condition

$$z(\xi, 0) = e^{\xi^2}.$$

We apply the LT method to Eq. (14) as

$$L\left[\frac{\partial^\beta z(\xi,t)}{\partial t^\beta}\right] = L\left[\frac{\partial^2 z(\xi,t)}{\partial \xi^2} - (2 + 4\xi^2 - 2t)z(\xi,t)\right],$$

$$s^\beta z(\xi,t) - s^{\beta-1} z(\xi,0) = L\left[\frac{\partial^2 z(\xi,t)}{\partial \xi^2} - (2 + 4\xi^2 - 2t)z(\xi,t)\right].$$

Therefore, according to inverse LT

$$z_0(\xi,t) = L^{-1}\left[\frac{z(\xi,0)}{s}\right],$$

$$z_{j+1}(\xi,t) = L^{-1}\frac{1}{s^\beta}\left[L\left[\frac{\partial^2 z_j(\xi,t)}{\partial \xi^2} - (2 + 4\xi^2 - 2t)z_j(\xi,t)\right]\right],$$

for $j = 0, 1, 2, \ldots$.

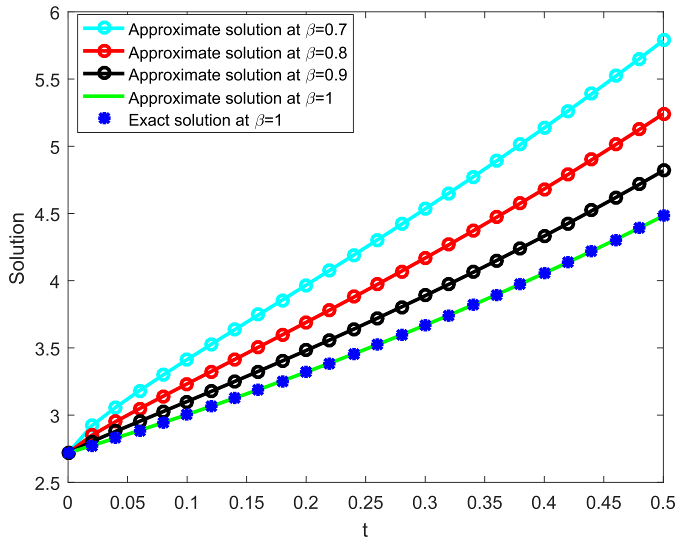

Fig. 4. Comparison of exact and LADM results of the Problem 4.2 at $\xi = 1$ against various values of t and β.

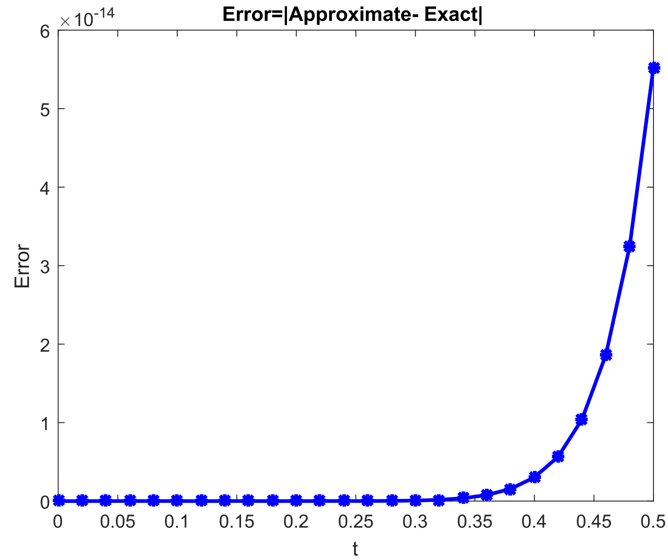

Fig. 6. Absolute error plot of LADM results of the Problem 4.2 against various values of t and $\beta = 1$.

We obtain

$$z_0(\xi, t) = e^{\xi^2},$$

$$z_1(\xi, t) = \frac{2e^{\xi^2} t^{\beta+1}}{\Gamma(\beta+2)},$$

$$z_2(\xi, t) = \frac{2^2(\beta+2)e^{\xi^2} t^{2(\beta+1)}}{\Gamma(2\beta+3)},$$

$$z_3(\xi, t) = \frac{2^3(\beta+2)(2\beta+3)e^{\xi^2} t^{3(\beta+1)}}{\Gamma(3\beta+4)}.$$

Similarly, we can find $z_4, z_5, \ldots$.

Hence, the series solution becomes

$$\tilde{z}(\xi, t) = e^{\xi^2}\left[1 + \frac{2t^{\beta+1}}{\Gamma(\beta+2)} + \frac{2^2(\beta+2)t^{2(\beta+1)}}{\Gamma(2\beta+3)} + \frac{2^3(\beta+2)(2\beta+3)t^{3(\beta+1)}}{\Gamma(3\beta+4)} + \ldots\right]. \tag{15}$$

When $\beta = 1$, then solution in Eq.(15) is transferred in the solution $\tilde{z}(\xi, t) = e^{\xi^2 + t^2}$,

which is the exact solution of the RDE of integer order as provided in [27,28].

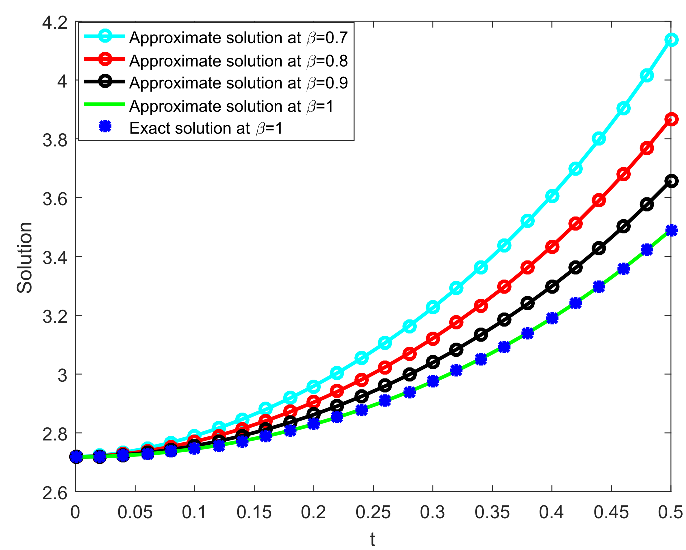

Fig. 7. Comparison of exact and LADM results of the Problem 4.3 at $\xi = 1$ at various values of t and β.

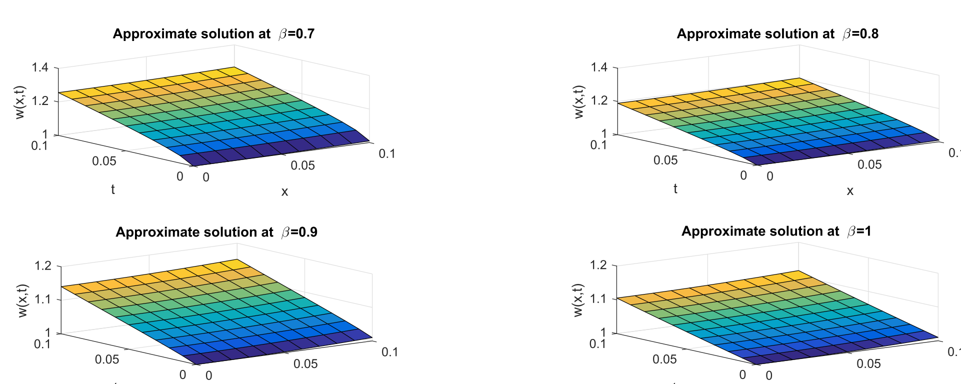

Fig. 5. LADM results of the Problem 4.2 at against values of $x(\xi)$, t and β.

Table 5
Results of Problem 4.3 by LADM against various value of the t at $\xi = 1$ and taking $\beta = 0.7, 0.8, 0.9$.

t	$LADM(\beta = 0.7)$	$LADM(\beta = 0.8)$	$LADM(\beta = 0.9)$
0	2.71828182846	2.71828182846	2.71828182846
0.04	2.73312373116	2.72818011041	2.7248581844
0.08	2.76688128388	2.75293336794	2.74291387866
0.12	2.81620183588	2.79075517308	2.77180587154
0.16	2.88027427114	2.84121235614	2.81145537499
0.20	2.95912229771	2.90438042967	2.86204560226
0.24	3.0532763561	2.98066783941	2.92395606779
0.28	3.16366443355	3.07075551358	2.99774152444
0.32	3.29157867182	3.17557811528	3.08412827752
0.36	3.43867905661	3.296326369	3.18402006405
0.40	3.60702090656	3.43446312627	3.29851072705
0.44	3.79910158286	3.59175047	3.42890272669
0.48	4.01792566985	3.77028720192	3.57673135879

Table 6
Absolute error of LADM results of Problem 4.3 at various values of the t at $\xi = 1$ and taking $\beta = 1$.

t	$Exact\,solution(\beta = 1)$	$LADMsolution(\beta = 1)$	$Error$
0	2.71828182846	2.71828182846	0
0.04	2.72263456064	2.72263456064	$6.94e-18$
0.08	2.73573462153	2.73573462153	0
0.12	2.75770827592	2.75770827592	$6.94e-18$
0.16	2.78876821962	2.78876821962	$6.94e-18$
0.20	2.82921701435	2.82921701435	$6.94e-18$
0.24	2.879452005	2.879452005	$6.94e-18$
0.28	2.93997183096	2.93997183096	0
0.32	3.01138468133	3.01138468133	$6.94e-18$
0.36	3.09441848514	3.09441848514	0
0.40	3.18993327612	3.18993327612	$6.94e-18$
0.44	3.2989360256	3.2989360256	0
0.48	3.42259830184	3.42259830184	0

For accuracy and simplicity of the LADM, truncating the solution in (15) at level $n = 12$. Numerical results of Example 4.3 are shown in Tables 5, 6 and have been plotted in Plots 7–9. The results in Table 6 and Fig. 7 (Green line shows approximate solution and blue dots line shows exact solution) provide the comparison of exact and LADM approximate solutions at $\beta = 1$. A surface graph of the solutions of Example 4.3 is plotted in Fig. 8, wherein for simple execution of the Matlab code, we have replaced $\tilde{z}(\xi, t)$ by $w(x, t)$. Each plot in the figures has the demonstration of physical behavior of the approximate solutions. Moreover, the absolute error are plotted in Fig. 9. They show close agrement between the analytical and approximate results.

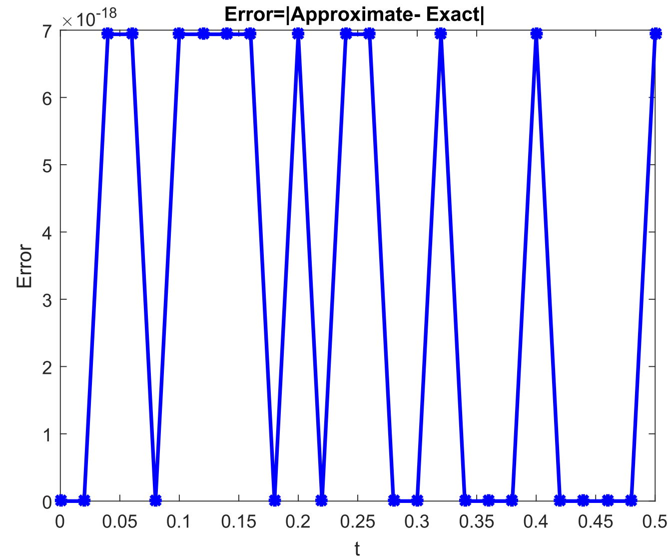

Fig. 9. Absolute error plot of LADM results of the Problem 4.3 at various values of t and $\beta = 1$.

Conclusion

In this research article, we have applied LADM to find the approximate solution of fractional order RDE. The concerned equations have great advantages in sciences and engineering. Further, the said equation constitutes more appropriate models for various physical systems in numerous areas such as spatial effects in biology, ecology and engineering. The LADM to fractional order RDE gives more realistic series solutions that converge very rapidly. It is noticeable that the LADM is less computational cost and consumes minimum time for treating FOPDEs. The main advantage of this method is its smooth convergence to the desired solution. The procedure of LADM is very simple, effective and accurate as observing the comparison of approximate solutions obtained via LADM to the exact solutions of problems. The LADM results also suggests that it can be used for other FOPDEs as well. All the computational works associated with problems in this research article are performed by using MATLAB.

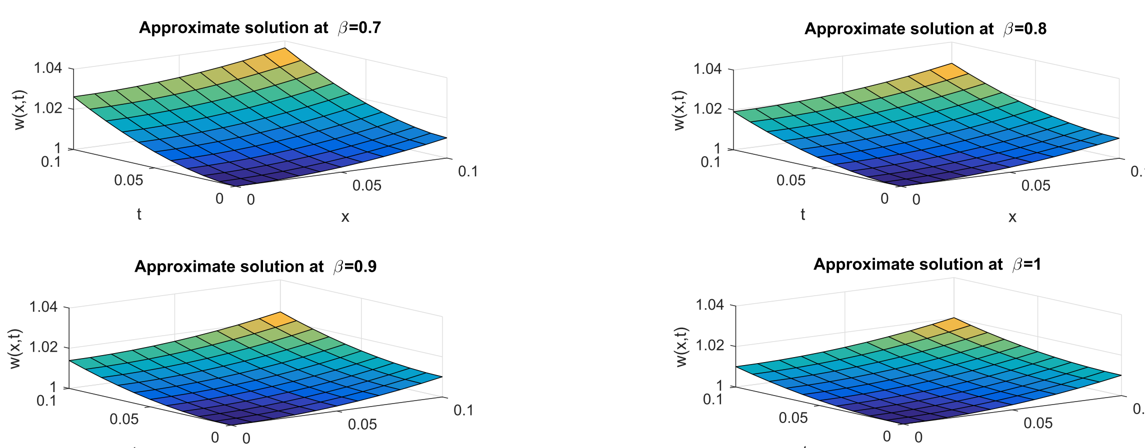

Fig. 8. LADM results of the Problem 4.3 against various values of $x(\xi), t$ and β.

Declaration of Competing Interest

None.

Compliance with Ethics Requirements

This article does not contain any studies with human or animal subjects.

Computation of Solution to Fractional Order Partial Cauchy Reaction Diffusion Equations.

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