Since January 2020 Elsevier has created a COVID-19 resource centre with free information in English and Mandarin on the novel coronavirus COVID-19. The COVID-19 resource centre is hosted on Elsevier Connect, the company's public news and information website.

Computer Methods and Programs in Biomedicine 196 (2020) 105642

Contents lists available at ScienceDirect

Computer Methods and Programs in Biomedicine

journal homepage: www.elsevier.com/locate/cmpb

Non Pharmaceutical Interventions for Optimal Control of COVID-19

Muhmmad Zamir[a], Zahir Shah[b,*], Fawad Nadeem[a], Arif Memood[c], Hussam Alrabaiah[d,e], Poom Kumam[f,g,**]

[a] Department of Mathematics, University of Science and Technology Bannu, Khyber Pakhtunkhwa, Pakistan
[b] Center of Excellence in Theoretical and Computational Science (TaCS-CoE), Science Laboratory Building, Faculty of Science, King Mongkut's University of Technology Thonburi (KMUTT), 126 Pracha-Uthit Road, Bang Mod, Thrung Khru, Bangkok 10140, Thailand
[c] Department of Mathematics and Statistics, Riphah International University Sector I-14, Islamabad, Pakistan
[d] College of Engineering, Al Ain University, Al Ain 64141, UAE
[e] Department of Mathematics, Tafila Technical University, Tafila 66110, Jordan
[f] KMUTT Fixed Point Research Laboratory, Room SCL 802 Fixed Point Laboratory, Science Laboratory Building, Department of Mathematics, Faculty of Science, King Mongkut's University of Technology Thonburi (KMUTT), Bangkok 10140, Thailand
[g] Department of Medical Research, China Medical University Hospital, China Medical University, Taichung 40402, Taiwan

ARTICLE INFO

Article history:
Received 18 May 2020
Accepted 29 June 2020

Keywords:
Novel coronavirus
Mathematical model
Basic reproduction number
Next generation matrix
Sensitivity analysis
Pontryagin's Maximum Principle
Optimal control

ABSTRACT

Background and Objective: The outbreak of the current pandemic begun from the first individual of a 55-year old from Hubei province in China, the disease instigated by the new coronavirus spreading across the world. Scientists presently speculate this coronavirus, SARS-CoV-2, originated in a bat and by one way or another jumped to another creature, potentially the pangolin, which at that point gave it to people. The ailment is currently spreading between individuals with no animal delegate. Researchers are struggling to follow the infection back to where it started to become familiar with its spread. In the event that, for example, specialists can locate the soonest cases, they might have the option to distinguish the creature have where the infection hides. In March and April 2020, researchers detailed that this virus created normally. Coronavirus has been become of the serious global phenomena in the recent years and has negative effects in the entire world health and economy. The virus is believed to have been associated with a host animal which human contracted. Subsequently, human-to-human infection began. Through migration as humans have become complex with easy mobility the disease has traveled to the entire continent. Now, numerous scientist are going on in the hope of obtaining medication and vaccination to prevent the spread of the disease and mortality of the disease. It is important that we obtain quantitative and qualitative information about the etiology of this disease which is crucial. Mathematical modeling is capable of providing qualitative information on many parameters that guides the decision making of health practitioners. In this work we focus the optimal control of COVID-19 with the help of Non Pharmaceutical Interventions (NPIs). To find the role of factors/parameters in the transmission of the syndrome we find R_0; the ratio of reproduction for the proposed model.

Methods: To find the role of parameters in the transmission of the syndrome we find R_0; the ratio of reproduction for the proposed model. On the basis of sensitivity indices of the parameters we apply Non Pharmaceutical Interventions(NPIs) to control the sensitive parameters and hence formulate the optimal control mode. With the help of Hamiltonian and Lagrangian we minimize the density of contaminated stuff and infected human population.

Results: We focus the optimal control of COVID-19 with the help of Non Pharmaceutical Interventions(NPIs). On the basis of sensitivity indices of the parameters we apply Non Pharmaceutical Interventions(NPIs) to control the sensitive parameters and hence formulate the optimal control model. The major NPIs are, *STAY HOME, SANITIZER (wash hands)*, *EARLY CASE DETECTION (PCR Test)* and *FACE MASK*. These NPIs helps in mitigation and reducing the size of outbreak of the disease.

Conclusion: We check the existence of the optimal solution for the system. At the end, Using matlab we produce numerical simulations for validation of results of control variables. The results demonstrate

* Corresponding author at:
** KMUTT Fixed Point Research Laboratory, Room SCL 802 Fixed Point Laboratory, Science Laboratory Building, Department of Mathematics, Faculty of Science, King Mongkut's University of Technology Thonburi (KMUTT), Bangkok 10140, Thailand.
E-mail addresses: zahir.sha@kmutt.ac.th (Z. Shah), poom.kum@kmutt.ac.th (P. Kumam).

https://doi.org/10.1016/j.cmpb.2020.105642

that if there is no control (variables/interventios), 900 out 1000 susceptible individuals may be infected (exposed) in very short period. As such a circumstances no agency fighting against COVID-19 could be successful due to its limited resources.

1. Introduction

Coronaviruses infact represents a big family. This family causes different types of infections. The infection ranges from common cold/flue to the most severe infection like severe acute respiratory syndrome and MERS; middle east respiratry syndrome [1]. In December, 2019, the human population of Wuhan, China was effected by severe cases of pneumonia and respiratory problems. The correct etiology of the infection could not be traced that time. WHO reported the virus as a novel coronavirus (2019-nCoV). The disease was named as severe acute respiratory syndrome coronavirus-2 (SARS-CoV-2). The virus was first identified from a single individual. Subsequently the virus was verified in sixteen more cases [2,3].

It is expected that the virus might be bat origin [4], and the infection transmission might be initiated from a seafood market (Huanan Seafood Wholesale Market) of China [5,8]. By April 27, 2020, the density of the infected population has reached 2,908,527 and 203,332 deaths, world wide [9].

The sings symptoms of COVID-19 includes, coughing, breath shortening and fever. The most severe attack of the virus may cause death. In some cases the attack may result SARS (severe acute respiratory syndrome) or pneumonia. The symptoms of infection appears in 2-14 days [6].

The coronavirus (2019-nCoV) is genetically related to the coronavirus that caused the SARS-2003, however the diseases they caused are quite different [7]. The genetic features and some clinical findings of the infection have been reported recently [8,10,11]. International air travel contributed the international spread of the infection. The infection has got global attention regarding its elimination and control[12].

The whole world is highly concerned with drastic future forecast of the disease. The scientists and researchers, therefore focus the development of mathematical model. The model not only helps estimating dynamics of the transmission of the virus but other important forecasts. Recent mathematical modeling includes [5,13–15]. These models mainly focused the spreading of coronavirus or basic reproduction number of coronavirus, (R_0). The authors followed intrinsic growth rate and the serial intervals. *Wu et al* in their study focus the forecasting and Newscasting of the novel coronavirus both nationally and internationally. The authors used Markov Chain Monte Carlo methods in their study [13].

However, these models don't the discuss the origin(bat) and the route of spreading (seafood market). *Chen et al* extended this work. The authors in their work presented a comprehensive model of novel coronavirus [16].

We, in this study focus the formulation of more comprehensive mathematical model of COVID-19. The proposed model incorporates hospitalized class, Quarantined class and non contaminated stuff class. These classes, so far the author information, were not included before. Also till date non of the authors has addressed the Optimal Control model of COVID-19. We find the initial rate of disease transmission, R_0 and conduct the sensitivity test of R_0. We find the most sensitive parameters, regarding transmission, of the proposed model. On the basis of sensitivity indexes we propose Non Pharmaceutical control variables and formulate the optimal control model. We use numerical simulations to test the efficiency of applied NPI's.

2. Model Formulation

The model incorporates human population and the stuff being open to the shedding of coronavirus. The human population is divided in seven sub-classes while the stuff is divided in two sub classes. The sub-classes of Human population are: susceptible humans class (S), the exposed humans class (E), symptomatic infected humans class (I_1), the quarantined class Q, the hospitalized class H, asymptomatic infected humans class (I_2), and recovered humans class (R). W_s; denote the stuff required by the human class for its livelihood. W_I denotes the surfaces/stuff shedded/stained with corona virus.

The susceptible human can catch infection from any of the infected class (E, I_1, I_2, H) and also from the contaminated stuff at different rates. A person who has contact with any of these infected classes is exposed to the attack of coronavirus. These exposed individuals are placed in infectious class I_1 if they develop sign symptoms of the disease and are quarantined (Q) otherwise. Some of the exposed individuals do not develop signs and symptoms and are silently infectious they are placed in class I_2. The infectious individuals with symptoms and those in quarrantina are shifted to hospital for proper facilitation Like PCR test and anti body tests. Most of the infected individual recover after 14 days however some of the infected individuals face problems in inhalation and are put on ventilators till recovery or death.

The mathematical model of the disease is given by the following set of coupled differential equations:

$$\begin{cases} \dot{S} = \Lambda - (\beta_1(I_1 + c_2 I_2 + c_3 H + c_4 E) + \beta_2 W_I)S - \mu S \\ \dot{E} = (\beta_1(I_1 + c_2 I_2 + c_3 H + c_4 E) + \beta_2 W_I)S - (\kappa_E + \mu)E \\ \dot{I_1} = r_1 \kappa_E E - (k_I + D_2 + \mu)I_1 \\ \dot{Q} = r_2 \kappa_E E - (k_\theta + \mu)Q \\ \dot{H} = k_I I_1 + k_\theta Q - (r_t + D_1 + \mu)H \\ \dot{R} = r_t H + r_n I_2 - \mu R \\ \dot{I_2} = (1 - (r_1 + r_2))\kappa_E E - (r_n + D_2 + \mu)I_2 \\ \dot{W_s} = A - (\xi_1 I_1 + \xi_2 I_2 + \xi_3 H)W_s - e_X W_s \\ \dot{W_I} = (\xi_1 I_1 + \xi_2 I_2 + \xi_3 H)W_s - (\varepsilon + e_X)W_I \end{cases} \quad (1)$$

The following Table (1) contains the values of the different parameters used in the model (1).

3. Model Analysis

Here, in this section 3 properties of the model; Disease Free Equilibrium, Invariant region and the Basic Reproduction Number are discussed.

3.1. Invariant Region

The time derivative of the state variables denote the change that occur in density of the concerned compartment and hence are taken non negative. Similar is the case of parameters. The total human population denoted by $\dot{N}$ and the total stuff denoted by $\dot{W}$ and are obtained by adding all the compartments of the concerned population. $\dot{N}$ and $\dot{W}$ are given by:

$$\dot{N} = \Lambda - \mu N - D_2(I_1 + I_2) - D_1 H, \quad (2)$$

Table 1
Table of the Values of parameters.

Notation	Parameter definition	Value	Source
Λ	Humans recruitment rate	0.0015875 day^{-1}	[22]
μ	Humans natural mortality rate	$0.00004day^{-1}$	[22]
c_2	transmission multiple of β_1 with I_2	$0.24666day^{-1}$	[17]
β_1	disease transmission from source I_1	$0.03day^{-1}$	[17]
c_3	transmission multiple of β_1 with H	$0.0103day^{-1}$	[17]
c_4	transmission multiple of β_1 with E	$0.31666day^{-1}$	[17]
κ_E	Inverse of disease incubation period	$0.142857day^{-1}$	[6]
k_I	disease clinical detection period	$0.1492537day^{-1}$	[17]
k_θ	Quarantine period	$0.071428day^{-1}$	[20]
β_2	disease transmission from contaminated stuff	$0.0147day^{-1}$	[18]
r_n	recovery rate without treatment	$0.035714day^{-1}$	[21]
r_t	recovery rate with treatment	$0.047619day^{-1}$	[21]
r_1	The ratio of exposed moving to I_1	0.25	[17]
r_2	The ratio of exposed moving to quarantine	0.16	[17]
ξ_1	shedding coefficient of I_1	$0.009day^{-1}$	assume
D_1	Disease induced death rate (with treatment)	$0.01day^{-1}$	[19]
ξ_2	shedding coefficient of I_2	$0.09day^{-1}$	assume
ξ_3	shedding coefficient of H	$0.07day^{-1}$	assume
D_2	Disease induced death rate (without treatment)	$0.012day^{-1}$	assume
A	Per capita stuff supply to market	$4 * \Gamma_h day^{-1}$	assume
ε	the life time of virus on stuff	$0.1day^{-1}$	[26]
e_X	stuff/food items expiry	$0.835day^{-1}$	assume

$$\dot{W} = A - e_X W - \varepsilon W_I, \tag{3}$$

From the above equations we have:

$$N \le N(0)e^{-\mu.t} + \frac{\Lambda}{\mu}\left(1 - e^{-\mu.t}\right) \Rightarrow \; N \le \frac{\Lambda}{\mu} \; when \; t \to \infty.$$

And

$$W \le W(0)e^{-e_X.t} + \frac{A}{e_X}\left(1 - e^{-e_X.t}\right) \Rightarrow \; W \le \frac{A}{e_X} \; when \; t \to \infty.$$

Using reference [23], we claim the following result:

Proposition 3.1. *Given the region Ω defined by*

$$\Omega = \left((S, E, I_1, Q, H, R, I_2, W_s, W_I) \in R_+^9 \, N \le \tfrac{\Lambda}{\mu}, W \le \tfrac{A}{e_X}\right).$$

is positively invariant domain, also the model is epidemiologically and mathematically well posed as all the trajectories are forward bounded.

3.2. Reproduction Number

The rate of spread of infection after its initial outbreak is called disease's Reproduction number. It is denoted by R_0 and is find by the following formula of generation matrix [22,24,25]. $R_0 = \rho(-FV^{-1})$, ρ being the spectral radius.

Here

$$f = \begin{pmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \\ f_5 \\ f_6 \end{pmatrix} = \begin{pmatrix} (\beta_1(I_1 + c_2 I_2 + c_3 H + c_4 E) + \beta_2 W_I)S \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

The column entries of matrix f denotes the newly infected individuals.

$F = \mathcal{J}_f$; the jacobian of the column matrix f.

$$F = \begin{pmatrix} a\beta_1 c_4 \frac{\Lambda}{\mu} & a\beta_1 \frac{\Lambda}{\mu} & 0 & a\beta_1 c_3 \frac{\Lambda}{\mu} & a\beta_1 c_2 \frac{\Lambda}{\mu} & \beta_2 \frac{\Lambda}{\mu} \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

and

$$v = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \end{pmatrix} = \begin{pmatrix} -(\kappa_E + \mu)E \\ r_1\kappa_E E - (k_I + D_2 + \mu)I_1 \\ r_2\kappa_E E - (k_\theta + \mu)Q \\ k_I I_1 + k_\theta Q - (r_t + D_1 + \mu)H \\ (1 - (r_1 + r_2))\kappa_E E - (r_n + D_2 + \mu)I_2 \\ (\xi_1 I_1 + \xi_2 I_2 + \xi_3 H)W_s - (\varepsilon + e_X)W_I \end{pmatrix}$$

The column entries of matrix v denotes the individuals that enter the infected class or leave the infected class, excluding those coming from susceptible class.

$V = \mathcal{J}_v$; the jacobian of the matrix v.

$$V = \begin{pmatrix} -(\kappa_E + \mu) & 0 & 0 & 0 & 0 & 0 \\ r_1\kappa_E & -(k_I + D_2 + \mu) & 0 & 0 & 0 & 0 \\ r_2\kappa_E & 0 & -(k_\theta + \mu) & 0 & 0 & 0 \\ 0 & k_I & k_\theta & -(r_t + D_1 + \mu) & 0 & 0 \\ (1 - (r_1 + r_2))\kappa_E & 0 & 0 & 0 & -(r_n + D_2 + \mu) & 0 \\ 0 & \xi_1 W_s & 0 & \xi_3 W_s & \xi_2 W_s & -(\varepsilon + e_X) \end{pmatrix}_{(DFE)}$$

Here

The dominant Eigenvalue of $(-FV^{-1})$ and hence R_0 is:

$$\begin{aligned} R_0 = {} & \frac{c_4 a\beta_1 \Lambda}{\mu(\kappa_E + \mu)} + \frac{r_1\kappa_E a\beta_1 \Lambda}{\mu(\kappa_E + \mu)(k_I + D_2 + \mu)} \\ & + \frac{(1 - (r_1 + r_2))\kappa_E a\beta_1 c_2 \Lambda}{\mu(\kappa_E + \mu)(r_n + D_2 + \mu)} + \frac{a\beta_1 c_3 \Lambda}{\mu(\kappa_E + \mu)(r_t + D_1 + \mu)} \\ & \left(\frac{r_1\kappa_E k_I}{(k_I + D_2 + \mu)} + \frac{r_2\kappa_E k_\theta}{(k_\theta + \mu)}\right) + \frac{\beta_2 \Lambda \kappa_E A}{e_X \mu(\kappa_E + \mu)(\varepsilon + e_X)} \\ & \left(\frac{(1 - (r_1 + r_2))\xi_2}{(r_n + D_2 + \mu)} + \frac{r_1\xi_1}{(k_I + D_2 + \mu)}\right. \\ & \left. + \frac{r_1 k_I \xi_3}{(k_I + D_2 + \mu)(r_t + D_1 + \mu)} + \frac{r_2 k_\theta \xi_3}{(k_\theta + \mu)(r_t + D_1 + \mu)}\right) \end{aligned}$$

Table 2
Sensitivity indices of parameters

Parameter	value	index	Parameter	value	index
κ_E	0.142857	1.9978	r_1	0.25	0.8347
β_1	0.03	0.8546	r_2	0.16	0.0077
c_2	0.24666	0.0027	ξ_1	0.009	0.0011
c_3	0.0103	0.000060828	ξ_2	0.09	0.0892
c_4	0.31666	0.0019	ξ_3	0.07	0.0551
Λ	0.0015875	0.1500	A	0.835	0.1454
ε	0.1	-0.1091	D_2	0.012	-0.0881
e_X	0.0333	-0.1817	β_2	0.0147	0.1454
μ	0.00004	-0.9999	D_1	0.01	-0.0096
r_t	0.047619	-0.0455	k_θ	0.071428	-0.0325
r_n	0.047619	-0.0687	k_I	0.1492537	-0.7535

3.3. Sensitivity analysis of R_0

The sensitivity of Z with respect to the parameter K is defined as [22].

$$\Upsilon_Z^K = \frac{\partial Z}{\partial K}\frac{K}{Z}.$$

The following Table (2) contains the sensitivities of the indices of the model is given below:

3.4. Control variables on the basis Sensitivity indices

To control the further transmission of the disease in the community, we use sensitivity test to check the absolute value of index of a parameter. Greatest the absolute value of the index of parameter, highest would be role of the parameter in disease transmission. All the parameters appearing in R_0 have impact on the inial rate of corona transmission. However the control of some of these parameters is beyond human control, like the incubation period etc.

The highest sensitivity indices are; κ_E = 1.9, β_1 = 0.85, μ = -0.9999, A= 0.14, β_2= 0.1454, k_I= - 0.75, e_X= -0.1817 and ε= - 0.1091.

κ_E is the incubation period of COVID-19 in human class. Though the sensitivity index of κ_E is high but the control of incubation period of a disease is beyond the human control. Therefore we can't intervene it. Similarly, μ, the natural death rate of human has got high sensitivity index and is inversely proportional to R_0. That is increase in μ by 10% would decrease R_0 by 9.9%. However this increase is impractical and we can't intervene. A, the stuff/food supply to market and e_X, the expiry period of food/stuff, effect the transmission of the disease. However this effect is not significant due to low sensitivity indices of the concerned parameters. Also this intervention would effect the supply and demand ratio of the market. Perhaps no government can afford to disturb the supply and demand ratio of the market.

β_1, the transmission probability of the infection from infected human to susceptible human, through direct contact or communication, has got the sensitivity of index 0.85. So a decrease of 50% in β_1 would reduce transmission rate by 42%. Control of β_1 directly effect the exposed class by reducing its density and it indirectly reduce the shedding coefficients of I_1 and I_2.

Case detection period, k_I, has got high sensitivity index and is inversely proportional to R_0. That is, quick case detection significantly reduce the transmission rate of disease. The sensitivity index of ε, the life time of virus, is low, however we introduce control variable, the sanitizer, here because sanitizer is more easily accessible intervention. Similar is the case of β_2, the chances of catching infection from contaminated stuff.

We address here the parameters, K_I; the detection period, β_1; social contacts, β_2; rate of catching infection from contaminated stuff/food products and ε; the life time of coronavirus. We introduce control variables h_1; stay home, h_2; use face mask and wash hands, h_3; testing labs, and h_4; sanitizer, to control the sensitive parameters of the model.

4. Optimal Control Model

Based on the above control variables we propose the following optimal control model.

$$\begin{cases}
\dot{S} = \Lambda - (\beta_1(1-h_1)(I_1 + c_2I_2 + c_3H + c_4E) + \beta_2(1-h_2)W_I)S - \mu S \\
\dot{E} = (\beta_1(1-h_1)(I_1 + c_2I_2 + c_3H + c_4E) + \beta_2(1-h_2)W_I)S - (\kappa_E + \mu)E \\
\dot{I}_1 = r_1\kappa_E E - (k_I(1+h_3) + D_2 + \mu)I_1 \\
\dot{Q} = r_2\kappa_E E - (k_\theta + \mu)Q \\
\dot{H} = (k_I(1+h_3)I_1 + k_\theta Q - (r_t + D_1 + \mu)H \\
\dot{R} = r_tH + r_nI_2 - \mu R \\
\dot{I}_2 = (1-(r_1+r_2))\kappa_E E - (r_n + D_2 + \mu)I_2 \\
\dot{W}_s = A - (\xi_1I_1 + \xi_2I_2 + \xi_3H)W_s - (e_X)W_s \\
\dot{W}_I = (\xi_1I_1 + \xi_2I_2 + \xi_3H)W_s - ((\varepsilon(1+h_4)) + e_X)W_I
\end{cases} \tag{4}$$

To minimize/reduce the density of infected human population and contaminated stuff we define an objective function as:

$$J(h_1,h_2,h_3,h_4) = \int_0^T (a_1I_1 + a_2I_2 + a_3W_I + \frac{1}{2}(g_1h_1^2(t) + g_2h_2^2(t) + g_3h_3^2(t) + g_4h_4^2(t)))dt \tag{5}$$

The weight constants of the symptomatic infectious human, I_1 and asymptomatic infectious human population, I_2 are a_1 and a_2. The weight constants of contaminated stuff is a_3.

The cost of different interventions in disease control is represented by the term:

$$\frac{1}{2}(g_1h_1^2(t) + g_2h_2^2(t) + g_3h_3^2(t) + g_4h_4^2).$$

Let us denote the control function by J_1 then we need to find $(h_1^*, h_2^*, h_3^*, h_4^*)$ so that

$$J_1 = J(h_1^*, h_2^*, h_3^*, h_4^*)$$

subject to the system (4), that is

$$J_1 = min\{J(h_1,h_2,h_3,h_4), (h_1,h_2,h_3,h_4) \in D\}.$$

D denoted the set of control variables and is defined as:

$$D = \{(h_1,h_2,h_3,h_4) \mid h_i(t) \text{ is lebesgue measurable on } [0,T], 0 \le h_i(t) < 1, i = 1,2,3,4\}$$

4.1. Solution of the proposed optimal control model

In this Section, we consider the control system (4) with initial conditions at $t = 0$ to investigate the existence of the control problem. For this we use [27] and the results there in.

We claim that the solution of the state system (4) is bounded. Because the control variable are bounded lebesgue measurable and the initial conditions are non-negative.

Next we prove that the system has optimal solution.

For this we introduce Lagrangian and Hamiltonian as:

$$L(t) = a_1 I_1 + a_2 I_2 + a_3 W_I + \frac{1}{2}(g_1 h_1^2 + g_2 h_2^2 + g_3 h_3^2 + g_4 h_4^2), \tag{6}$$

and

$$\mathbb{H}(t) = L(t) + \ell_1 \frac{dS}{dt} + \ell_2 \frac{dE}{dt} + \ell_3 \frac{dI_1}{dt} + \ell_4 \frac{dQ}{dt} + \ell_5 \frac{dH}{dt} + \ell_6 \frac{dR}{dt} + \ell_7 \frac{dI_2}{dt} + \ell_8 \frac{dW_s}{dt} + \ell_9 \frac{dW_I}{dt} \tag{7}$$

where $\mathbb{H}(t)$ is meant for the minimal value of the Lagrangian.

Theorem 4.1. *The set $(h_1^*, h_2^*, h_3^*, h_4^*)$ of optimal control, minimize J over D, subject to the initial conditions specified at $t = 0$.*

Proof. Since both state and control variables are non-negative. So using [28], the objective function is convex in control variables $h_i,\ i = 1, 2, 3, 4$.

Clearly from def, D is closed and convex.

Further, since the optimal system is bounded hence the set of optimal control is compact. And the control variables h_1, h_2, h_3 and h_4 occur as quadratic. Therefore the integrand in (5) is convex on control set D.

Also, all Y, for $Y \in \Omega$, are bounded. So We can find a constant $\partial > 1$ and positive numbers ω_1 and ω_2 such that

$$J(h_1, h_2, h_3, h_4) \geq \omega_1(|h_1|^2, |h_2|^2, |h_3|^2, |h_4|^2)^{\frac{\partial}{2}} - \omega_2.$$

Which completes the proof of existence of an optimal control. □

Next we characterize the control variables of the proposed model. We use Pontryagin Maximum Principle [29]; stated here for convenience. Let

$M = (S^*, E^*, I_1^*, Q^*, H^*, R^*, I_2^*, W_s^*, W_I^*)^T$ be the states associated with control variables $(h_1^*, h_2^*, h_3^*, h_4^*)$.

Let (y, h) be the solution of (4), for $y \in \Omega$ and $h = (h_1^*, h_2^*, h_3^*, h_4^*)$ then $\exists$ some vector function

$$\ell = (\ell_1, \ell_2, ..., \ell_n)$$

so that:

$$\frac{dy}{dt} = \frac{\partial \mathbb{H}(t, y, h, \ell)}{\partial \ell}$$

$$0 = \frac{\partial \mathbb{H}(t, y, h, \ell)}{\partial h}$$

$$\ell' = -\frac{\partial \mathbb{H}(t, y, h, \ell)}{\partial y}$$

For necessary conditions on $\mathbb{H}$ we prove the following result.

Theorem 4.2. *The necessary condition for control $h = (h_1^*, h_2^*, h_3^*, h_4^*)$ to be optimal with corresponding state M is that $\exists$ adjoint variables:*

ℓ_i *for* $i = 1, 2, ..., 9$ *satisfying;*

$$\begin{cases}
\frac{d\ell_1}{dt} = (\ell_1(t) - \ell_2(t))((1 - h_1(t))\beta_1(I_1 + c_2 I_2 + c_3 H + c_4 E) \\
\quad + \beta_2 W_I(1 - h_2(t))) + \ell_1 \mu \\
\frac{d\ell_2}{dt} = (\ell_1(t) - \ell_2(t))(\beta_1 S c_4(1 - h_1(t)) + (\ell_7(t) - \ell_3(t))\kappa_E r_1 \\
\quad + (\ell_7(t) - \ell_4(t))\kappa_E r_2 + (\ell_2(t) - \ell_7(t))\kappa_E + \ell_2(t)\mu \\
\frac{d\ell_3}{dt} = (\ell_1(t) - \ell 2(t))(\beta_1 S(1 - h_1(t)) + (\ell_3(t) - \ell_5(t))(K_I - h_3) \\
\quad + (\ell_8(t) - \ell_9(t))W_S \xi_1 + \ell_3(D_2 + \mu) - a_1 \\
\frac{d\ell_4}{dt} = (\ell_4(t) - \ell_5(t))K_\theta + \ell_4(t)\mu \\
\frac{d\ell_5}{dt} = (\ell_1(t) - \ell_2(t))(\beta_1 S c_3(1 - u_1(t)) \\
\quad + (\ell_8(t) - \ell_9(t))W_S \xi_3 + (\ell_5(t) - \ell_6)r_t + \ell_5(t)(D_1 + \mu) \\
\frac{d\ell_6}{dt} = \ell_6 \mu \\
\frac{d\ell_7}{dt} = (\ell_1(t) - \ell_2(t))\beta_1 c_2 S(1 - h_1(t)) + (\ell_7(t) \\
\quad - \ell_6(t))r_n + (\ell_8(t) - \ell_9(t))W_S \xi_2 + \\
\ell_7(D_2 + \mu) - a_2 \\
\frac{d\ell_8}{dt} = (\ell_8(t) - \ell_9(t))(\xi_1 I_1 + \xi_1 I_2 \\
\quad + \xi_1 H) + \ell_8 e_x - a_3 \\
\frac{d\ell_9}{dt} = (\ell_1(t) - \ell_2(t))\beta_2 S(1 - h_2(t)) \\
\quad + \ell_9(t)(\varepsilon + e_x + h_4 + \mu) - a_3
\end{cases} \tag{8}$$

with transversality conditions or boundary conditions

$$\ell_i(t_{end}) = \ell_i(T) = 0 \ \text{for}\ i = 1, 2, ..., 9. \tag{9}$$

And the optimal controls h_1^*, h_2^*, h_3^* and h_4^* are given by

$$h_1^* = max\left(min\left\{(\ell_1 - \ell_2)\frac{S\beta_1(I_1 + c_2 I_2 + c_3 H + c_4 E)}{g_1}, 1\right\}, 0\right) \tag{10}$$

$$h_2^* = max\left(min\left\{(\ell_1 - \ell_2)\frac{W_I \beta_2 S}{g_2}, 1\right\}, 0\right) \tag{11}$$

$$h_3^* = max\left(min\left\{(\ell_5 - \ell_3)\frac{I_1}{g_3}, 1\right\}, 0\right) \tag{12}$$

$$h_4^* = max\left(min\left\{\frac{(\ell_9 W_I)}{g_4}, 1\right\}, 0\right) \tag{13}$$

Proof. First we differentiate $\mathbb{H}$ given in (7), with respect to the state variables involved in the model. As a result we get the system $\frac{d\ell_i}{dt}$ for i=1,2,3...9.

To evaluate $(h_1^*, h_2^*, h_3^*, h_4^*)$, we solve the following system, using properties of control set and optimality conditions.

$$\frac{\partial \mathbb{H}}{\partial h_1} = 0, \quad \frac{\partial \mathbb{H}}{\partial h_2} = 0, \quad \frac{\partial \mathbb{H}}{\partial h_3} = 0, \quad \frac{\partial \mathbb{H}}{\partial h_4} = 0$$

Solving this system we obtain the results shown in (10)–(13). □

Here, the second derivative of Lagrangian L with respect to h_1^*, h_2^*, h_3^* and h_4^* is $+ve$. This show that the optimal control is maximum at control h_1^*, h_2^*, h_3^* and h_4^*.

We put these values in system (4) and propose the following optimal control model.

$$\begin{cases}
\dot{S^*} = \Lambda - ((1 - h_1^*)\beta_1(I_1^* + c_2 I_2^* + c_3 H^* + c_4 E^*) \\
\quad + \beta_2(1 - h_2^*)W_I^*)S^* - \mu S^* \\
\dot{E^*} = ((1 - h_1^*)\beta_1(I_1^* + c_2 I_2^* + c_3 H^* + c_4 E^*) \\
\quad + \beta_2(1 - h_2^*)W_I^*)S^* - (\kappa_E + \mu)E^* \\
\dot{I_1^*} = r_1 \kappa_E E^* - (k_I(1 + h_3^*) + D_2 + \mu)I_1^* \\
\dot{Q^*} = r_2 \kappa_E E^* - (k_\theta + \mu)Q^* \\
\dot{H^*} = (k_I(1 + h_3^*)I_1^* + k_\theta Q^* - (r_t + D_1 + \mu)H^* \\
\dot{R^*} = r_t H^* + r_n I_2^* - \mu R^* \\
\dot{I_2^*} = (1 - (r_1 + r_2))\kappa_E E^* - (r_n + D_2 + \mu)I_2^* \\
\dot{W_s^*} = A - (\xi_1 I_1^* + \xi_2 I_2^* + \xi_3 H^*)W_s^* - (e_X + h_4^*)W_s^* \\
\dot{W_I^*} = (\xi_1 I_1^* + \xi_2 I_2^* + \xi_3 H^*)W_s^* - (\varepsilon(1 + h_4^*) + e_X)W_I^*
\end{cases} \tag{14}$$

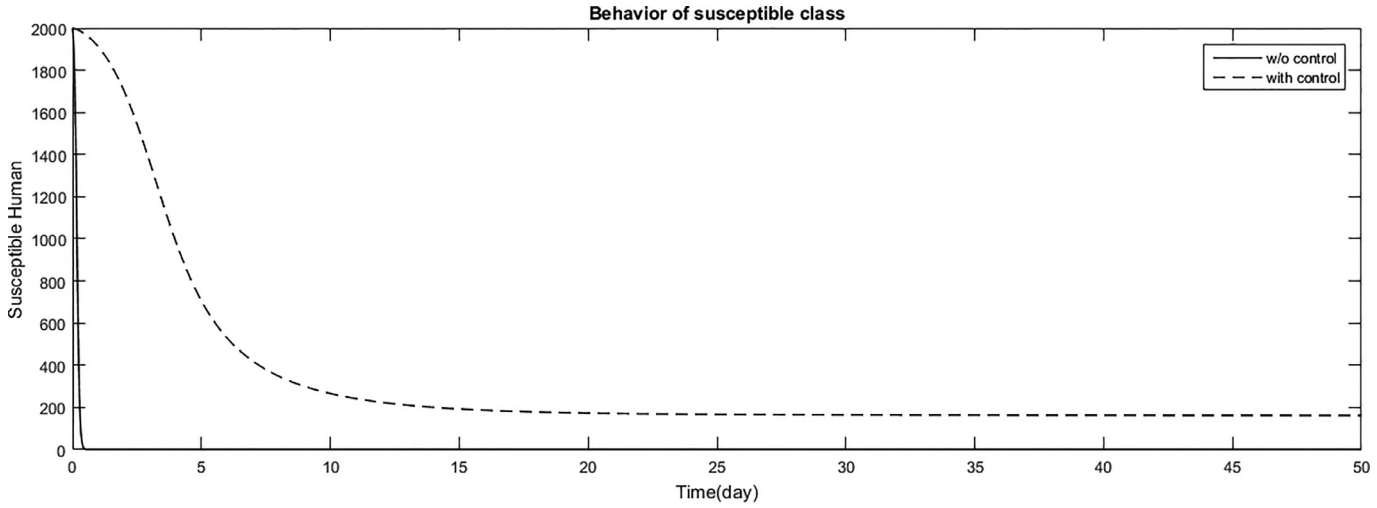

Fig. 1. The fig represents the effect of interventions/control variables on susceptible population.

with

$$\begin{aligned}\mathbb{H}^* &= a_1 I_1^* + a_2 I_2^* + a_3 W_l^* \\ &+\frac{1}{2}\Big(g_1\Big(maximum\Big\{min\Big\{(\ell_1-\ell_2)\frac{S\beta_1(I_1+c_2I_2+c_3H+c_4E)}{g_1},1\Big\},0\Big\}\Big)^2 \\ &+g_2\Big(maximum\Big\{min\Big\{(\ell_1-\ell_2)\frac{W_l\beta_2 S}{g_2},1\Big\},0\Big\}\Big)^2 \\ &+g_3\Big(maximum\Big\{min\Big\{(\ell_5-\ell_3)\frac{I_1}{g_3},1\Big\},0\Big\}\Big)^2 \\ &+g_4\Big(maximum\Big\{min\Big\{\frac{(\ell_9 W_l)}{g_4},1\Big\},0\Big\}\Big)^2\Big)+\ell_1\frac{dS^*}{dt}+\ell_2\frac{dE^*}{dt}+\ell_3\frac{dI_1^*}{dt} \\ &+\ell_4\frac{dQ_l^*}{dt}+\ell_5\frac{dH^*}{dt}+\ell_6\frac{dR^*}{dt}+\ell_7\frac{dI_2^*}{dt}+\ell_8\frac{dW_s^*}{dt}+\ell_9\frac{dW_l^*}{dt} \end{aligned} \tag{15}$$

5. Numerical simulations

We solve the optimality system equation (14), numerically using RK4 method. The method solve the state system (4), forward in time and the adjoint system (8), backward in the time and the system (9), the controls are updated continuously using system (10) to (13). The process is continued until the results at the consecutive iterations are too closed. For detail [30]. we assign the following values to weight constants. $a_1 = 0.0181$, $a_2 = 0.010$, $a_3 = 0.1$, $g_1 = 0.081$, $g_2 = 0.01$, $g_3 = 0.1$ and $g_4 = 0.008$. We have used $S = 2000$, $E = 10$, $I_1 = 10$, $Q = 5$, $H = 5$, $R = 5$, $I_2 = 15$, $W_s = 4000$ and $W_l = 100$.

In the following figures we present the effect of proposed non pharmaceutical interventions. Since the initial rate of disease transmission, R_0, is very high. Therefore the density of susceptible population reduces very soon in the initial period of few days. And in the same period the density of exposed class reaches the peak as shown in Figs. (1) and (2). The figures show that with in small period of 5-10 days, 1800 out 2000 susceptible humans catch infection. The proposed non pharmaceutical interventions helps in control and reduction of disease but in particular it helps to flatten the curve of infection. Curve flattening means to reduce the patient burden in the hospitals and quarantine centers as shown in Figs. (3)–(5). Fig. (8) shows that recovery with out control is high than that of with control. The reason is that our control variables are non pharmaceutical and therefore recovery is infact a ratio. Thats is about 80% to 95% of total infected individuals recover. So greater the number of infected individuals, greater would be the number of recoveries. Face mask and sanitizer are not only useful in self defence but also play better role in control of density of contami-

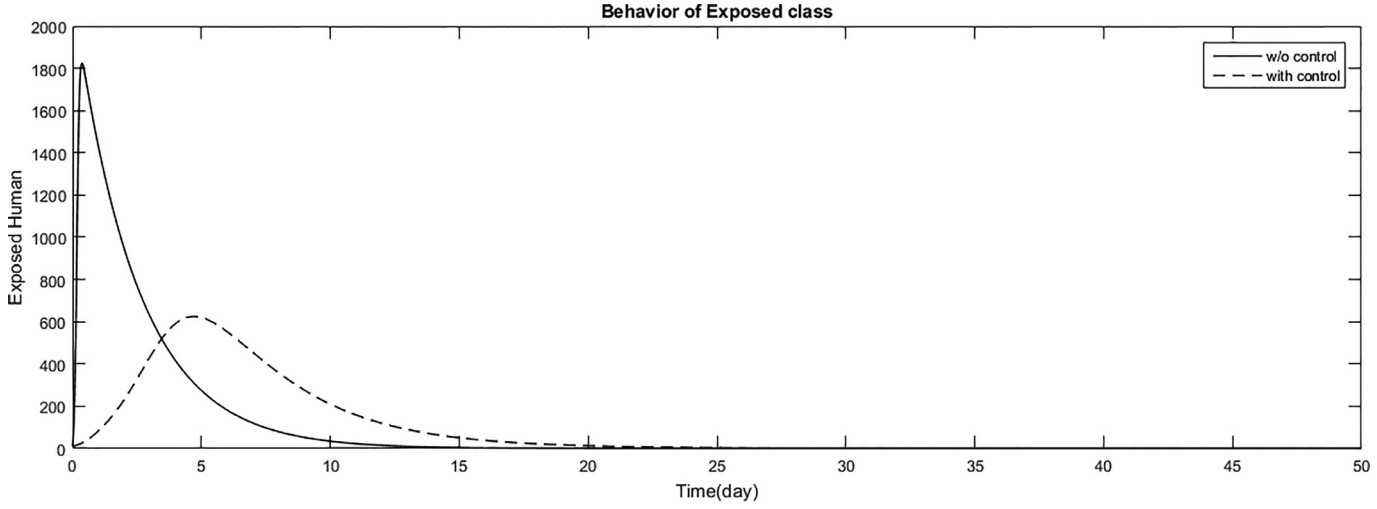

Fig. 2. The behavior of exposed class with and without control. The gap between the graphs denotes the effect of interventions.

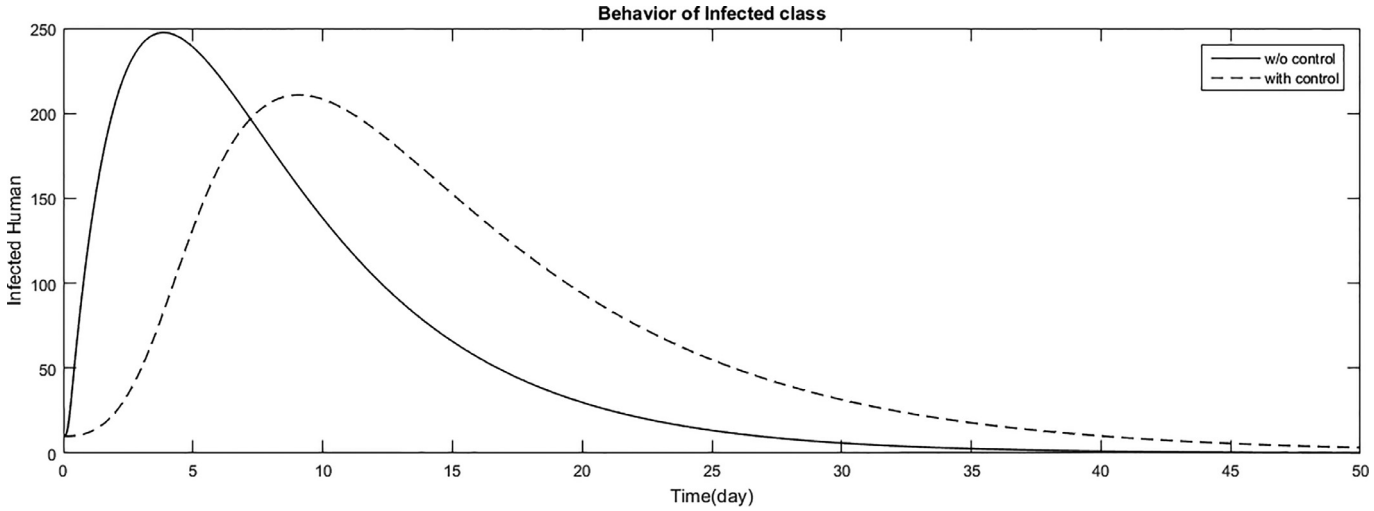

Fig. 3. The effect of interventions on the density of symptomatic infectious class.

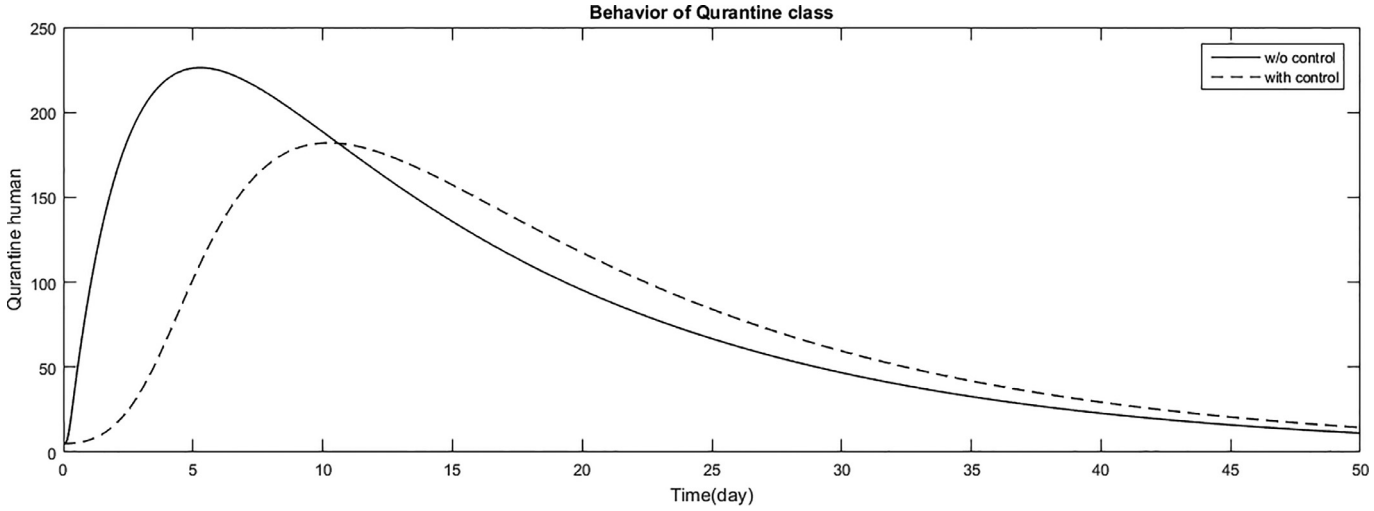

Fig. 4. The fig represents the density of the quarantine class with and without control variables.

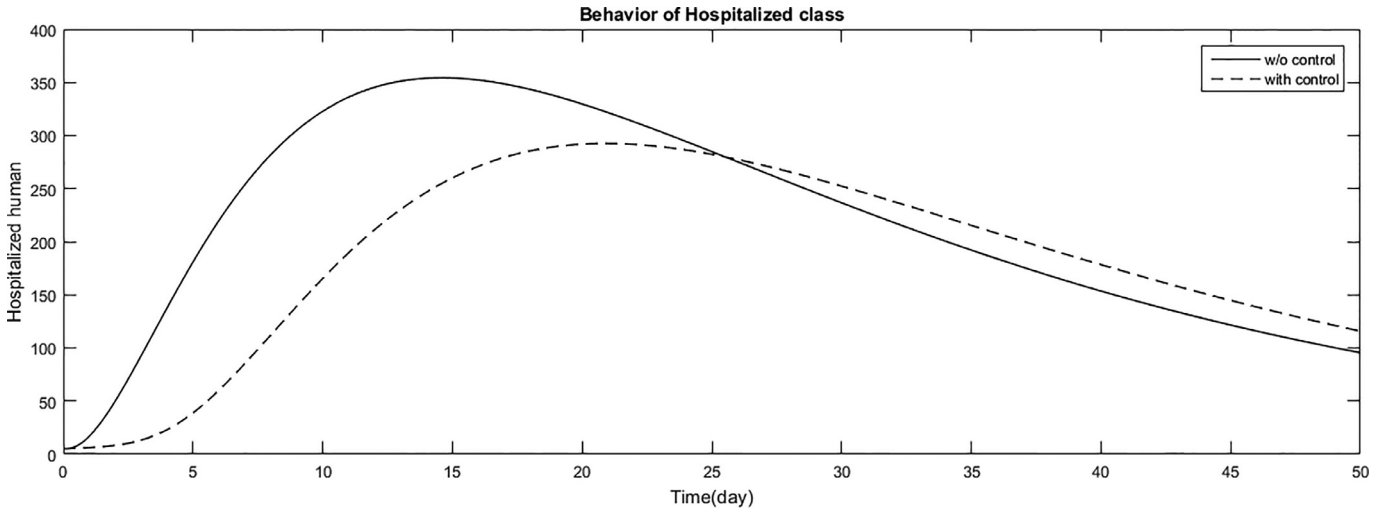

Fig. 5. The fig represents the burden of patients in hospitals with and without control variables.

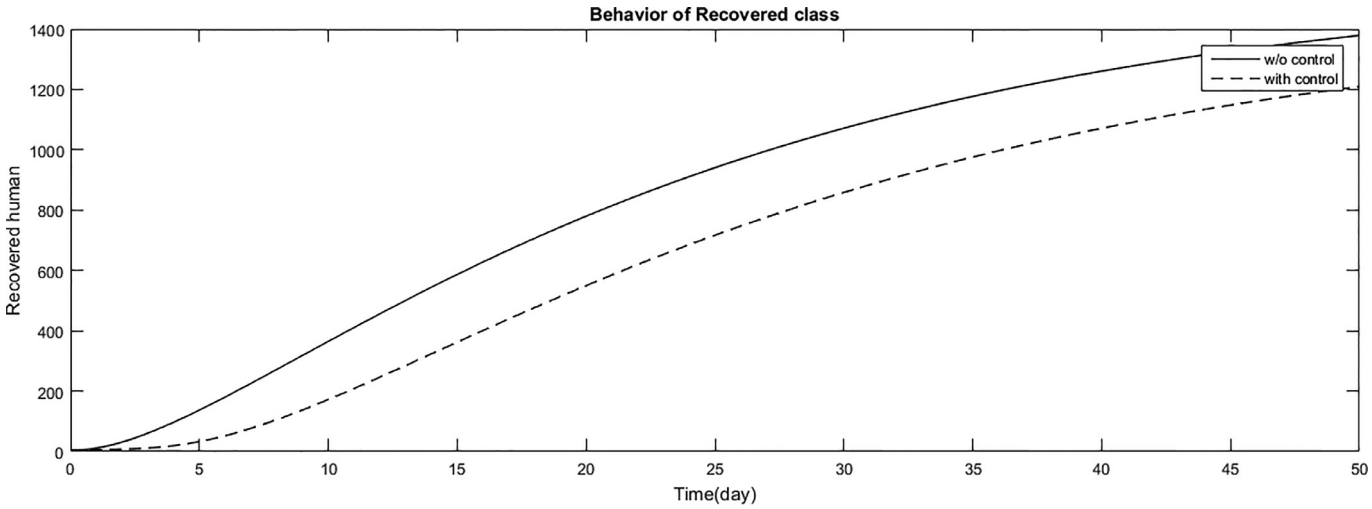

Fig. 6. The fig represents the recovered human population with and without interventions.

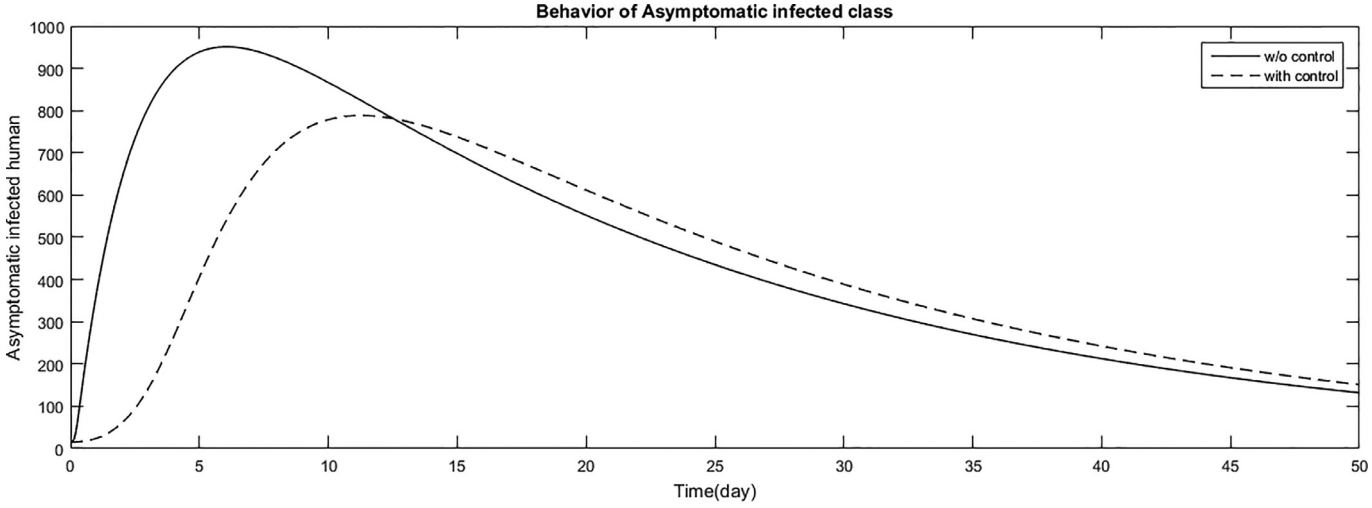

Fig. 7. The fig represents the effect of interventions on asymptomatic human class. The difference of the graphs denotes the effectiveness of control variables.

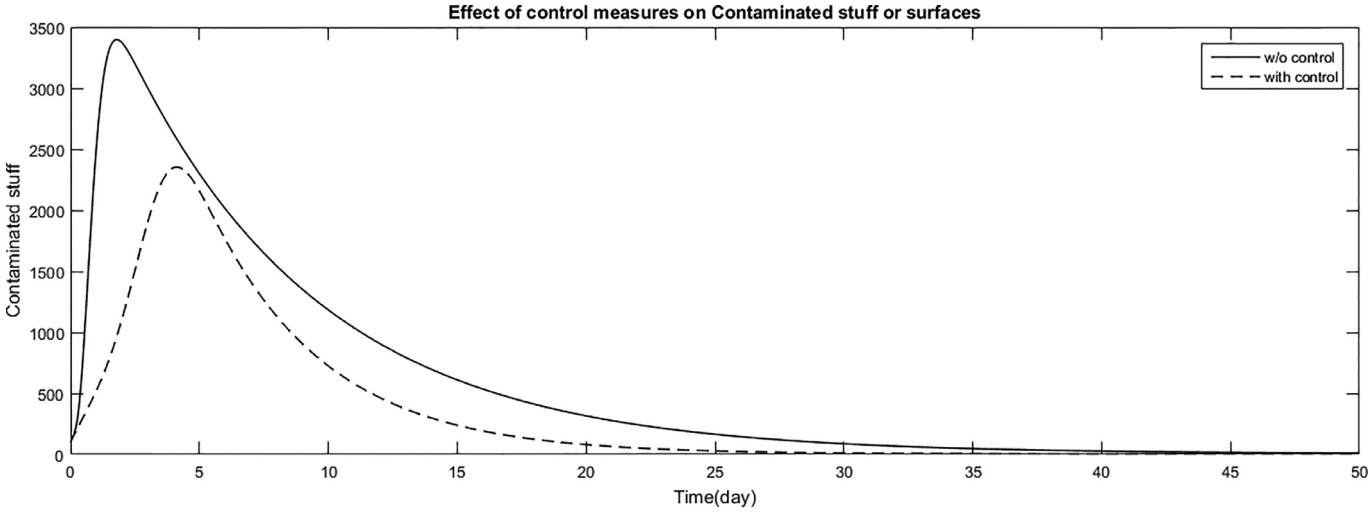

Fig. 8. The fig represents the effect of interventions upon the density of contaminated stuff.

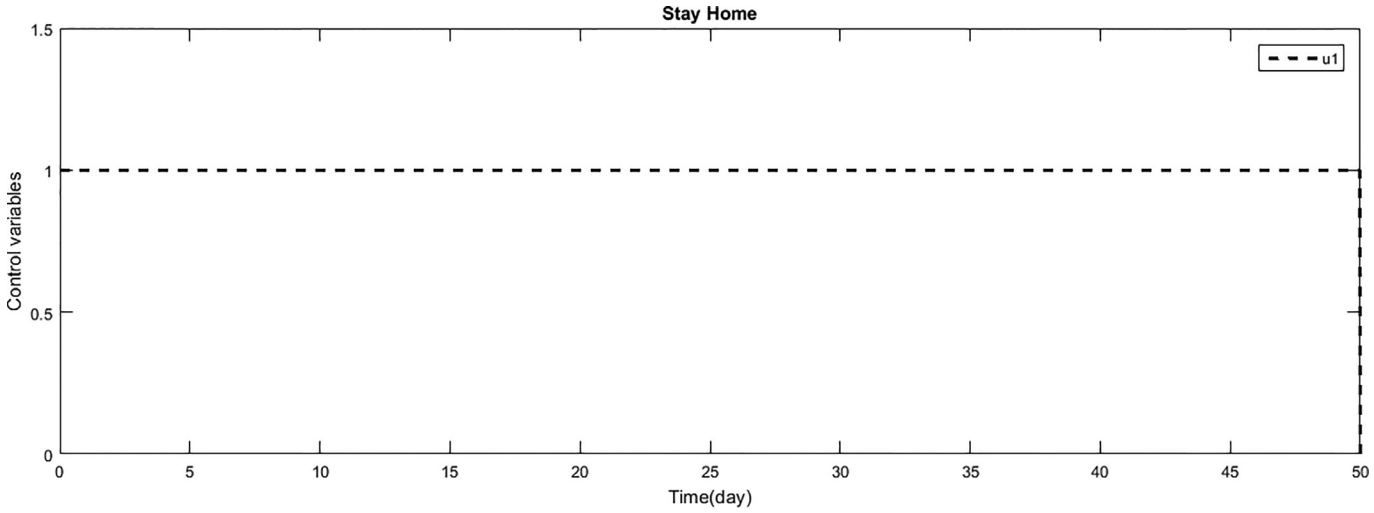

Fig. 9. The fig represents the intervention 'STAY HOME'.

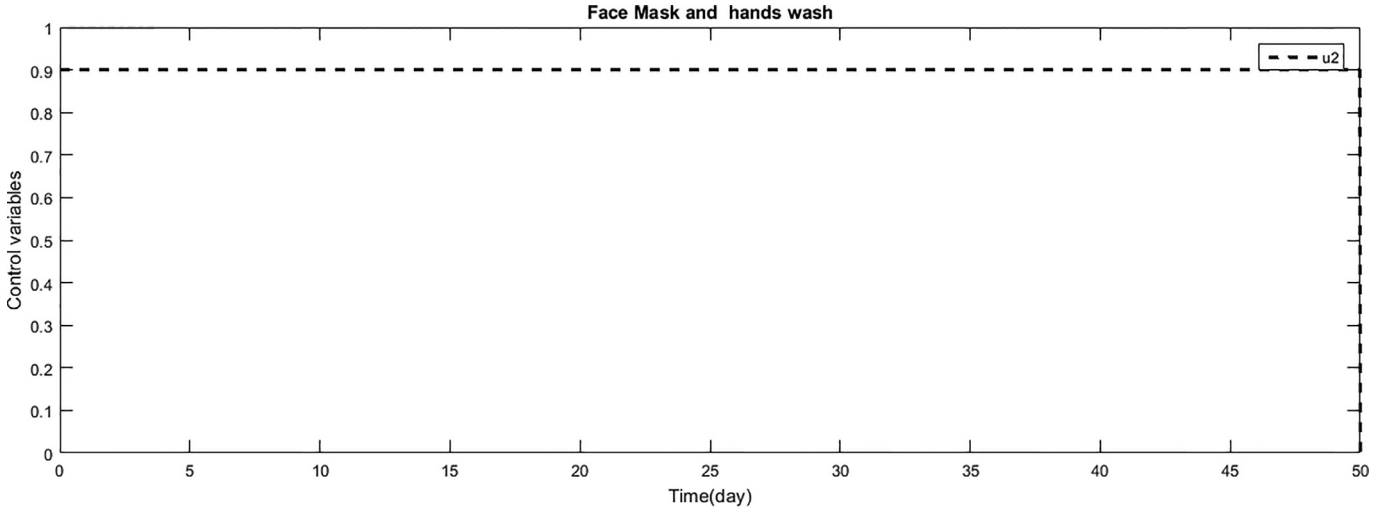

Fig. 10. The fig represents the intervention 'FACE MASK'.

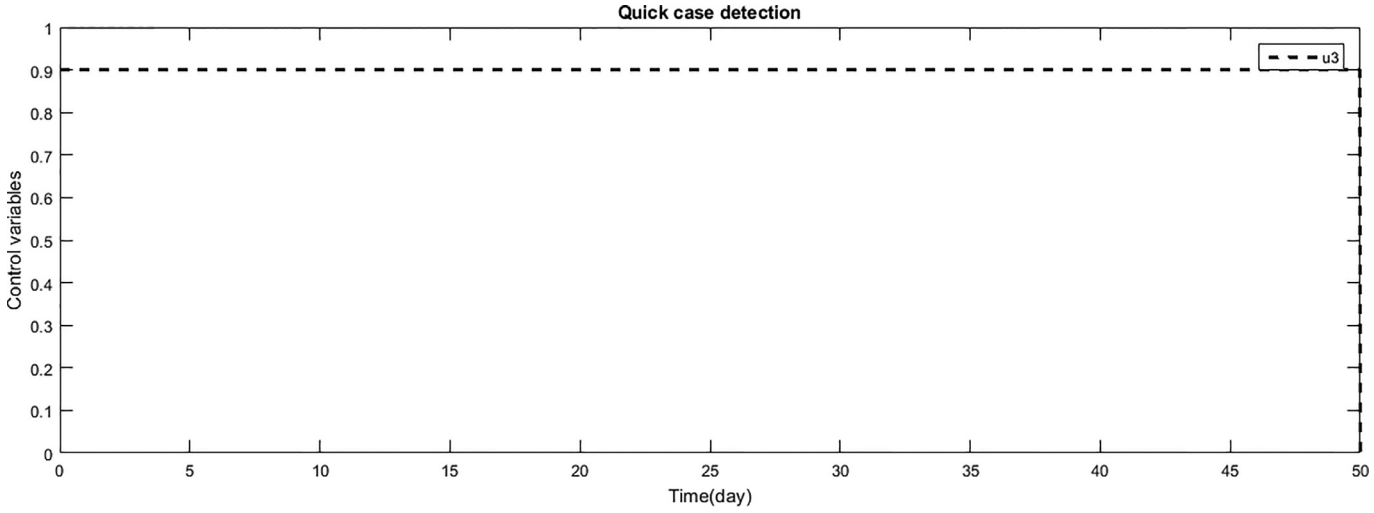

Fig. 11. The fig represents the intervention 'QUICK CASE DETECTION' with help of laboratory tests.

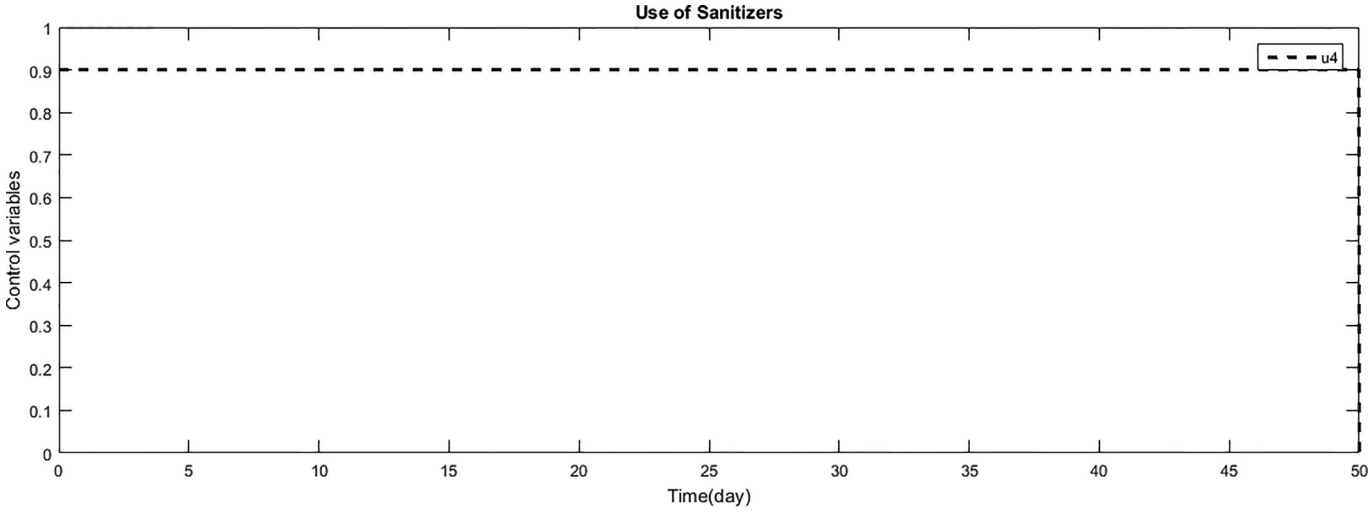

Fig. 12. The fig represents the intervention 'REGULAR USE OF SANITIZER'.

nated stuff by reducing the shedding coefficients ξ_1, ξ_2 and ξ_3, as shown in Fig. (6).

6. Conclusion

In this work we tried to seek the optimal control of COVID-19 pandemic. The sensitivity test of the reproduction number shows that non of the parameters and particularly easily commendable parameters, have got the dominant role in disease transmission. Therefore the control of this disease is comparably tough but not disappointing. We have introduced four control variables in the transmission concerned parameters; β_1, the transmission probability of the infection from infected human to susceptible human, Case detection period, k_I, ε, the life time of virus and β_2, the contact with non-living contaminated items.

The disease is not so dangerous in the sense that it takes about 14 days in its recovery from mild cases and 30-40 days in severe cases with disease induced mortality rate 1% to 4%. However the disease transmission rate is no doubt alarming as shown by the Fig. (2). The figure demonstrate that if there is no control (variables/interventios), 1800 out 2000 susceptible individuals may be infected (exposed) in very short period. As such a circumstances no agency fighting against COVID-19 could be successful due to its limited resources. The same is the problem with hospitalized and quarantined class as shown in Figs. (4) and (5). No hospital nor quarantined center can accommodate such a huge number of infected individuals.

However the implementation of all the strategies(control variables, h_2, h_3 and h_4) at the same time for at least 50 days may esuriently reduce the number of new cases. Figs. (2), (5) and (4) in the presence of active disease interventions the density of exposed (infected) individual will be reduced from 1800 to 650 individuals and consequently reducing the burden of hospitalization to 260 and quarantine to 170 only. Also considerable decrease is observed in the contaminated class as shown in Fig. (8).

It is worth mentioning to keep the non pharmaceutical interventions continue till the elimination of the pandemic. Otherwise the new outbreaks will occur, and due to highest rate of disease transmission, will infect the community once again.

Declaration of Competing Interest

The authors declare that they have no competing interests

Acknowledgments

"This research is supported by Postdoctoral Fellowship from King Mongkut's University of Technology Thonburi (KMUTT), Thailand".

References

[1] Myth busters. Available: https://www.who.intnovel-coronavirus-2019advice-for-public.
[2] WHO statement regarding cluster of pneumonia cases in wuhan, china. Available: https://www.who.int/china/news/detail/09-01-2020.
[3] World health organization. coronavirus. world health organization, cited january 19, 2020. Available: https://www.who.int/health-topics/coronavirus.
[4] P. Zhou, X.L. Yang, X.G. Wang, B. Hu, L. Zhang, W. Zhang, A pneumonia outbreak associated with a new coronavirus of probable bat origin, Nature (2020). https://doi.org/10.1038/s41586-020-2012-7
[5] Q. Li, X. Guan, P. Wu, X. Wang, L. Zhou, Y. Tong, Early transmission dynamics in wuhan, china, of novel coronavirus-infected pneumonia, N Engl J Med (2020). https://doi.org/10.1056/NEJMoa2001316
[6] Coronavirus poster https://www.who.intdocscoronavirus-poster-english-srilanka.
[7] Q and a on coronaviruses (COVID-19) https://www.who.intnews-roomq-a-detailq-a-coronaviruses.
[8] C. Huang, Y. Wang, X. Li, L. Ren, J. Zhao, Y. Hu, Clinical features of patients infected with 2019 novel coronavirus in wuhan, china, Lancet (2020). https://doi.org/10.1016/S0140-6736(20)30183-5
[9] COVID-19 CORONAVIRUS / CASES. Available at: https://www.worldometers.info/coronavirus/coronavirus-cases/.
[10] J.F. Chan, S. Yuan, K.H. Kok, K.K. To, H. Chu, J. Yang, A familial cluster of pneumonia associated with the 2019 novel coronavirus indicating person-to-person transmission: a study of a family cluster, Lancet (2020). https://doi.org/10.1016/S0140-6736(20)30154-9
[11] N. Zhu, D. Zhang, W. Wang, X. Li, B. Yang, J. Song, A novel coronavirus from patients with pneumonia in china, 2019, N Engl J Med (2020). https://doi.org/10.1056/NEJMoa2001017
[12] I.I. Bogoch, A. Watts, A. Thomas-Bachli, C. Huber, M.U.G. Kraemer, K. Khan, Pneumonia of unknown etiology in wuhan, china: Potential for international spread via commercial air travel, J Travel Med. (2020). https://doi.org/10.1093/jtm/taaa008
[13] J.T. Wu, K. Leung, G.M. Leung, Nowcasting and forecasting the potential domestic and international spread of the 2019-ncov outbreak originating in wuhan, china: a modelling study, Lancet (2020). https://doi.org/10.1016/S0140-6736(20)30260-9
[14] S. Zhao, Q. Lin, J. Ran, S.S. Musa, G. Yang, W. Wang, Preliminary estimation of the basic reproduction number of novel coronavirus (2019-ncov) in china, from 2019 to 2020: a data-driven analysis in the early phase of the outbreak, Int J Infect Dis (2020). https://doi.org/10.1016/j.ijid.2020.01.050
[15] S. Zhao, S.S. Musa, Q. Lin, J. Ran, G. Yang, W. Wang, Estimating the unreported number of novel coronavirus (2019-ncov) cases in china in the first half of january 2020: a data-driven modelling analysis of the early outbreak, J Clin Med (2020). https://doi.org/10.3390/jcm9020388
[16] T.M. Chen, J. Rui, Q.P. Wang, Z.Y. Zhao, J.A. Cui, L. Yin, Amathematical model for simulating the phase-based transmissibility of a novel coronavirus, Infectious Diseases of Poverty (2020). https://doi.org/10.1186/s40249-020-00640-3
[17] B. Ivorra, M.R. Ferrandez, M. Vela-Perez, A.M. Ramos, Mathematical modeling of the spread of the coronavirus disease 2019, (COVID-19) considering its par-

ticular characteristics, The case of China, MOMAT (2020) 1–23, doi:10.13140/RG.2.2.21543.29604.
[18] C. Yang, J. Wang, A mathematical model for the novel coronavirus epidemic in wuhan, china, Math. Biosci. Eng. 17 (3) (2020) 2708–2724.
[19] J.A. Spencer, D.P. Shutt, S.K. Moser, H. Clegg, H.J. Wearing, H. Mukundan, et al., Epidemiological parameter review and comparative dynamics of influenza, respiratory syncytial virus,rhinovirus, human coronvirus, and adenovirus, medrxiv, 2020.
[20] Coronavirus-why 14 days quarentine period. Available on: https://www.health24.com/Medical/Infectious-diseases/Coronavirus/coronavirus-why-the-14-day-quarantine-period-20200311-3.
[21] Report of the WHO-china joint mission on coronavirus disease 2019 (COVID-19.). Available on: https://www.who.int/docs/default-source/coronaviruse/who-china-joint-mission-on-covid-19-final-report.pdf.
[22] F. Nadeem, M. Zamir, A. Tridane, Y. khan, Modeling and control of zoonotic cutaneous leishmaniasis, Journal of Mathematics 51 (2) (2019) 105–121. ISSN 1016-2526
[23] M. Zamir, G. Zaman, A.S. Alshomrani, Sensitivity analysis and optimal control of anthroponotic cutaneous leishmania, PloS one (2016) 1–19, doi:10.1371/journal.pone.0160513.
[24] M. Zamir, G. Zaman, A.S. Alshomrani, Control strategies and sensitivity analysis of anthroponotic visceral leishmaniasis model, J. Biol. Dyna. 11 (1) (2017) 323–338.
[25] M. Zamir, R. Sultana, R. Ali, W. Panhwar, S. Kumar, Study on the threshold conditions for infection of visceral leishmaniasis, Sindh Univ. Res. J. (Sci. Ser.) 47 (3) (2015) 619622.
[26] C. Geller, M. Varbanov, R.E. Duval, Human coronaviruses: Insights into environmental resistance and its influence on the development of new antiseptic strategies, Viruses 4 (2012) 30443068.
[27] G. Birkhoff, G.C. Rota, Ordinary Differential Equations, fourth, John Wiley and Sons, New York, 1989.
[28] D.L. Lukes, Differential equations: Classical to controlled, Mathematics in Science and Engineering, volume 162, Academic Press, New York, 1982.
[29] M.I. Kamien, N.L. Schwartz, Dynamics optimization: The calculus of variations and optimal control in economics and management, 1991.
[30] W.K. Hackbusch, A numerical method for solving parabolic equations with opposite orientations, Computing 20 (1978).