Alexandria Engineering Journal (2021) 60, 3669–3678

HOSTED BY

ELSEVIER

Alexandria University

Alexandria Engineering Journal

www.elsevier.com/locate/aej
www.sciencedirect.com

Dynamical analysis of fractional-order tobacco smoking model containing snuffing class

Hussam Alrabaiah [a,b,*], Anwar Zeb [c], Ebraheem Alzahrani [d], Kamal Shah [e]

[a] College of Engineering, Al Ain University, Al Ain, United Arab Emirates
[b] Department of Mathematics, Tafila Technical University, Tafila, Jordan
[c] Department of Mathematics, COMSATS University Islamabad, Abbottabad Campus, Khyber Pakhtunkhwa, Pakistan
[d] Department of Mathematics, Faculty of Science, King Abdulaziz University, P.O. Box 80203, Jeddah 21589, Saudi Arabia
[e] Department of Mathematics, Malakand University, Chakdara, Dir, Pakistan

Received 1 January 2021; revised 1 February 2021; accepted 3 February 2021
Available online 2 March 2021

KEYWORDS

Fractional calculus;
Fractional systems;
Dynamics of model;
GABMM;
Computational work

Abstract The current pandemic situation caused by COVID-19 has affected human life globally at the economic, social and mental health levels. Specifically, tension has led an increasing number of people to the consumption of various types of tobacco. In this work, an existing tobacco smoking model with a specific class of tobacco snuffing is converted into a fractional order as many applications of fractional derivatives to recall the past history of smokers in the present model. For this purpose, we use fractional derivative in Caputo sense to study the model in the form of fractional order. Then Positivity, boundness and dynamics of the proposed model are investigated. For numerical results, the generalized "Adams–Bashforth–Moulton Method (GABMM) and fourth-order Runge–Kutta (RK4) method" are used to solve the proposed model and Matlab numerical computing environment is the current software used.

1. Introduction

Mathematical models in fractional order have proven to be useful in manifesting wide range of phenomena mathematically than integer-order models because of fractional order reveals the past history and hereditary properties in models especially in the infectious diseases models. Mathematicians usually use, for simplicity, ordinary differential equations in integer order. In applied sciences, mathematical modeling has found widespread applications and in particular the fractional calculus, see [1–17]. Smoking is a cause of many diseases including many type of cancers In current pandemic of COVID-19 virus, smokers are at more risk to be affected by COVID-19 - because of many reasons including of smokers' fingers are in touch with their lips regularly during smoking and this habit leads to increase the probability of transmission of virus from hand to mouth. Worldwide, those people who are smoking suffered increasingly from different disease like cancer of lungs, lips, throat. In this way the immune system

* Corresponding author at: College of Engineering, Al Ain University, Al Ain, UAE.
E-mail addresses: hussam.alrabaiah@aau.ac.ae (H. Alrabaiah), anwar@cuiatd.edu.pk (A. Zeb), eoalzahrani@kau.edu.sa (E. Alzahrani).
Peer review under responsibility of Faculty of Engineering, Alexandria University.

https://doi.org/10.1016/j.aej.2021.02.005
1110-0168

of smokers people weakens due to which they are easily exposed to serious disease like corona virus disease. Cigarette smokers are 2 to 4 times more likely to get heart disease than nonsmokers and also doubles a person's risk for stroke and also higher risk to caught lung cancer. To increase the life expectancy of humans, scientists, doctors and mathematicians have tried to control smoking through modelling that contains media or education campaign or in the form of anti-nicotine medicine [18–31]. Mathematicians have tried to make different smoking models to represent cigarette smoking phenomena. This work was initiated by Castillo–Garsow et al. [18] in their model where they discussed the potential smokers represented by P, smokers represented by S, and quit smokers represented by Q. Then a modified model of smoking that contained chain smokers class was presented by Sharami et al. [19]. Recently, researchers have designed several smoking models under various linear, saturated, square-root-type and harmonic-mean-type incidence rates [22,25–27,21,29–31]. Nowadays, researchers attempt to bring about different fractional order epidemic models. Due to a lot of applications, fractional calculus is applied in different scientific fields [32–38]. This research work demonstrates the smoking model in fractional order with snuffing class and determine the existence of an analytical and numerical solution of our proposed model, presented in [30] as:

$$
\begin{aligned}
\frac{dX}{dt} &= \lambda - \beta_1 X H_1 - \mu X + \alpha Y,\\
\frac{dH_1}{dt} &= \beta_1 X H_1 - \beta_2 H_1 H_2 - (\rho + \mu) H_1,\\
\frac{dH_2}{dt} &= \beta_2 H_1 H_2 - (d + \omega + \mu) H_2,\\
\frac{dY}{dt} &= \omega H_2 - (\alpha + \gamma + \mu) Y,\\
\frac{dZ}{dt} &= \gamma Y - \mu Z,
\end{aligned}
\tag{1}
$$

under the initial conditions:

$$
X(0) = e_1,\ H_1(0) = e_2,\ H_2(0) = e_3,\ Y(0) = e_4,\ Z(0) = e_5,
\tag{2}
$$

for the parameters description see Table 1, of [30].nd

To include the past history or hereditary properties in our model, we establish the fractional order derivatives instead of integer order derivatives in system (1). As the term $Z(t)$ does not appear in the first four equations of system (1), therefore without loss of generality, we can take out $Z(t)$ from system (1). So, the following set of differential equations in fractional order can be written as a new system:

$$
\begin{aligned}
{}_0^C D_t^\alpha X(t) &= \lambda - \beta_1 X H_1 - \mu X + \alpha Y,\\
{}_0^C D_t^\alpha H_1(t) &= \beta_1 X H_1 - \beta_2 H_1 H_2 - (\rho + \mu) H_1,\\
{}_0^C D_t^\alpha H_2(t) &= \beta_2 H_1 H_2 - (d + \omega + \mu) H_2,\\
{}_0^C D_t^\alpha Y(t) &= \omega H_2 - (\alpha + \gamma + \mu) Y.
\end{aligned}
\tag{3}
$$

Here, the notation D_t^α stands for derivative in Caputo sense with order $0 < \alpha \leq 1$. The fractional order system is converted to ordinary differential equations system when $\alpha = 1$. System (3) leads to generalization for system (1). As integer-order epidemic models have established fruitful understanding for biological systems, more realistic biological models memory and after-effect properties are presented by fractional order models, especially in smoking dynamics. Therefore, the fractional-order derivatives are applied on system (1). The stability of the system is discussed as the same as proved in [30]. For basics of fractional calculus and fractional order differential equations (FODEs) see [39–45].

The arrangement of the rest of the paper is given in Section 2, where the dynamics of the fractional order model is presented. The intention of GABMM is presented in Sections 3, with a brief introduction for solution of fractional order smoking model. Section 4 is devoted to numerical simulation results of the GABMM, where comparisons of results obtained with GABMM and Runge-Kutta method (RKM) taken place in Section 5. Last section is devoted to a brief conclusion.

2. Dynamics of the fractional order model

Here, this part we derive results about positivity and boundedness. We define space by

$\mathscr{R}_+^4 = \{(X, H_1, H_2, Y) | X, H_1, H_2, Y \geqslant 0\}$.

Theorem 1. *Let* $(X_0, H_{10}, H_{20}, Y_0) \in \mathbb{R}_+^4$ *is initial values and* $(X(t), H_1(t), H_2(t), Y(t))$ *be any solution. Then, the set* $\mathscr{R}_+^4$ *is a positively invariant. Also one has*

$$
\begin{aligned}
\limsup_{t\to\infty} X(t) &\leqslant X_\infty := \tfrac{\lambda + \alpha Y_\infty}{\mu},\\
\limsup_{t\to\infty} H_1(t) &\leqslant H_{1\infty} := \tfrac{\lambda}{(\rho+\mu)},\\
\limsup_{t\to\infty} H_2(t) &\leqslant H_{2\infty} := \tfrac{\lambda}{(d+\omega+\mu)},\\
\limsup_{t\to\infty} Y(t) &\leqslant Y_\infty := \tfrac{\omega H_{2\infty}}{(\alpha+\gamma+\mu)}.
\end{aligned}
\tag{4}
$$

Proof. For the model (3), we have

$$
\begin{aligned}
{}_0^C D_t^\alpha X|_{X=0} &= \lambda + \alpha Y > 0,\\
{}_0^C D_t^\alpha H_1|_{H_1=0} &= 0,\\
{}_0^C D_t^\alpha H_2|_{H_2=0} &= 0,\\
{}_0^C D_t^\alpha Y|_{Y=0} &= \omega H_2 \geqslant 0.
\end{aligned}
\tag{5}
$$

Upon using generalized mean value theorem [46,47] together with (5), one has $X(t), H_1(t), H_2(t), Y(t) \geqslant 0$, for all values of $t \geqslant 0$. Equation first of the system (3) yields

$${}_0^C D_t^\alpha X \leqslant \lambda - \mu X + \alpha Y_\infty.$$

$2nd$ and $3rd$ equations of the system (3) implies that

$${}_0^C D_t^\alpha (X + H_1 + H_2) \leqslant \lambda - \mu X + \alpha Y_\infty - (\rho + \mu) H_1 - (d + \omega + \mu) H_2,$$

which yields

$$\limsup_{t\to\infty} [X(t) + H_1(t) + H_2(t)] \leqslant H_{1\infty}$$

and

$$\limsup_{t\to\infty} [X(t) + H_1(t) + H_2(t)] \leqslant H_{2\infty}.$$

Accordingly, it follows the second and third estimate of (4). Now by the last equation of system (3), one has

$${}_0^C D_t^\alpha Y \leqslant \omega H_{2\infty} - \alpha Y - \mu Y - \gamma Y$$

for large enough t. Which leads the fourth estimate of (4). □

2.1. The reproduction number and equilibrium points

Solving the following algebraic equations for finding the equilibria of the model (3),

$$\begin{aligned} \lambda - \beta_1 X H_1 - \mu X + \alpha Y &= 0, \\ \beta_1 X H_1 - \beta_2 H_1 H_2 - (\rho + \mu) H_1 &= 0, \\ \beta_2 H_1 H_2 - (d + \omega + \mu) H_2 &= 0, \\ \omega H_2 - (\alpha + \gamma + \mu) Y &= 0. \end{aligned} \tag{6}$$

Two solutions to the system (6) are obtained via using some algebraic manipulations as $E_0 = \left(\frac{\lambda}{\mu}, 0, 0, 0\right)$, and $E^* = \left(X^*, H_1^*, H_2^*, Y^*\right)$, where

$$\begin{aligned} X^* &= \tfrac{\beta_2 H_2^* + (\rho^\alpha + \mu)}{\beta_1}, \\ H_1^* &= \tfrac{(d+\omega+\mu)}{\beta_2}, \\ H_2^* &= \tfrac{\omega H_2^*}{(\alpha+\gamma+\mu)}, \\ Y^* &= \tfrac{(\alpha+\gamma+\mu)(\rho+\mu)[\beta_2 \mu (R_0 - 1) - \beta_1 (d+\omega+\mu)]}{(\gamma+\mu)(\beta_1 \beta_2 \omega) + (\alpha+\gamma+\mu)(\beta_1 \beta_2 (d+\mu) + \beta_2^{2\alpha} \mu)}. \end{aligned}$$

The Jacobian of system (2) is

$$J = \begin{pmatrix} -\beta_1 H_1 - \mu & -\beta_1 X & 0 & \alpha \\ \beta_1 H_1 & \beta_1 X - \beta_2 H_2 - (\rho + \mu) & -\beta_2 H_1 & 0 \\ 0 & \beta_2 H_2 & \beta_2 H_1 - (d + \omega + \mu) & 0 \\ 0 & 0 & \omega & -(\alpha + \gamma + \mu) \end{pmatrix}.$$

Also at free equilibrium point E_0, the Jacobian is provided as

$$J(E_0) = \begin{pmatrix} -\mu & \frac{-\beta_1 \lambda}{\mu} & 0 & \alpha \\ 0 & \frac{\beta_1 \lambda}{\mu} - (\rho + \mu) & 0 & 0 \\ 0 & 0 & -(d + \omega + \mu) & 0 \\ 0 & 0 & \omega^\alpha & -(\alpha + \gamma + \mu) \end{pmatrix}.$$

Considering the given matrices to compute reproductive number

$$F = \begin{pmatrix} \frac{\beta_1 \lambda}{\mu} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

$$V = \begin{pmatrix} (\rho + \mu) & 0 & 0 \\ 0 & (d + \omega + \mu) & 0 \\ 0 & -\omega & (\alpha + \gamma + \mu) \end{pmatrix}.$$

The maximum eigenvalue of FV^{-1} is $\frac{\beta_1 \lambda}{\mu(\rho+\mu)}$, so

$$R_0 = \frac{\beta_1 \lambda}{\mu(\rho + \mu)} \tag{7}$$

is the required reproductive number.

Theorem 2. *Under the condition $R_0 < 1$, then the system (3) is locally stable and if $R_0 > 1$, then system (3) is unstable.*

Proof. At E_0, the condition for local stability at the Jacobian of system (3) is given by

$$J(E_0) = \begin{pmatrix} -\mu & \frac{-\beta_1 \lambda}{\mu} & 0 & \alpha \\ 0 & \frac{\beta_1 \lambda}{\mu} - (\rho + \mu) & 0 & 0 \\ 0 & 0 & -(d + \omega + \mu) & 0 \\ 0 & 0 & \omega & -(\alpha + \gamma + \mu) \end{pmatrix},$$

which follows the eignvalues $\lambda_1,\ \lambda_2,\ \lambda_3$ and λ_4 as

$$\begin{aligned} \lambda_1 &= -\mu < 0, \\ \lambda_3 &= -(d + \omega + \mu) < 0, \\ \lambda_4 &= -(\alpha + \gamma + \mu) < 0, \\ \lambda_2 &= (\rho + \mu)(R_0 - 1), \end{aligned}$$

implies that $\lambda_2 < 0$ if $R_0 < 1, \lambda_2 = 0$ if $R_0 = 1$ and $\lambda_2 > 0$ if $R_0 > 1$. □

Theorem 3. *If $R_0 < 1$, then the system (3) is globally stable.*

Proof. For proof of this theorem see [30]. □

3. The generalized Adams–Bashforth–Moulton method

Here GABMM is presented in this section [48,49]. In this algorithm, the GABMM is derived for getting the numerical solution of the nonlinear FODEs. Let

$$Dy(t) = f(t, y(t)),\ 0 \leqslant t \leqslant T, \tag{8}$$

with

$$y^{(k)}(0) = y_0^k,\ k = 0, 1, \ldots [\alpha] - 1 \tag{9}$$

be a general problem of FODEs. We obtain the solution $y(t)$ in view of application of fractional integral on both sides of Eq. (8)

$$y(t) = \sum_{k=0}^{[\alpha]-1} \frac{y_0^{(k)}}{k!} t^k + \int_0^t \frac{(t-\tau)^{\alpha-1}}{\Gamma(\alpha)} f(\tau, y(\tau)) d\tau. \tag{10}$$

By setting $h = \frac{T}{m}, t_n = nh, n = 0, 1, \ldots, m,$ Eq. (10) can be described as follows for some positive integer m

$$\begin{aligned} y_h(t_{n+1}) = &\sum_{k=0}^{[\alpha]-1} \frac{y_0^k}{k!} t_{n+1}^k + \frac{h^\alpha}{\Gamma(\alpha+2)} f\big(t_{n+1}, y_h^p(t_{n+1})\big) \\ &+ \frac{h^\alpha}{\Gamma(\alpha+2)} \sum_{j=0}^{n} a_{j,n+1} f\big(t_j, y_h(t_j)\big). \end{aligned} \tag{11}$$

$$a_{j,n+1} = \begin{cases} n^{\alpha+1} - (n+1)^\alpha (n-\alpha), & \text{if j = 0}, \\ (n-j+2)^{\alpha+1} - 2(n-j+1)^\alpha + (n-j)^{\alpha+1}, & \text{if } 0 < \text{j} \leqslant \text{n}, \\ 1, & \text{if j = n + 1}, \end{cases}$$

In which the predicted value $y_h^p(t_{n+1})$ may be derived as

$$y_h^p(t_{n+1}) = \sum_{k=0}^{[\alpha]-1} \frac{y_0^k}{k!} t_{n+1}^k + \frac{1}{\Gamma(\alpha)} \sum_{j=0}^{n} b_{j,n+1} f\big(t_j, y_h(t_j)\big), \tag{12}$$

in which

$$b_{j,n+1} = \frac{h^\alpha[(n-j+1)^\alpha - (n-j)^\alpha]}{\alpha}.$$

The estimated error is

$$\max_{j=0,1,\ldots,m} |y(t_j) - y_h(t_j)| = 0(h^p),$$

in which $p = \min\{1 + \alpha, 2\}$.

4. Implementation of numerical simulation

Current part is related to numerical solution of the nonlinear fractional model using the GABM method. The numerical scheme of model (3) with the help of GABMM is given as follows:

$$X_h(t_{n+1}) = X_0 + \frac{h^\alpha}{\Gamma(\alpha+2)}\Big[f_1\big(t_{n+1}, X_h^p(t_{n+1}), H_{1h}^p(t_{n+1}), H_{2h}^p(t_{n+1}), Y_h^p(t_{n+1})\big) + \sum_{j=0}^{n} a_{j,n+1} f_1\big(t_j, X_h(t_j), H_{1h}(t_j), H_{2h}(t_j), Y_h(t_j)\big)\Big],$$

$$H_{1h}(t_{n+1}) = H_{10} + \frac{h^\alpha}{\Gamma(\alpha+2)}\Big[f_2\big(t_{n+1}, X_h^p(t_{n+1}), H_{1h}^p(t_{n+1}), H_{2h}^p(t_{n+1}), Y_h^p(t_{n+1})\big) + \sum_{j=0}^{n} a_{j,n+1} f_2\big(t_j, X_h(t_j), H_{1h}(t_j), H_{2h}(t_j), Y_h(t_j)\big)\Big],$$

$$H_{2h}(t_{n+1}) = H_{20} + \frac{h^\alpha}{\Gamma(\alpha+2)}\Big[f_3\big(t_{n+1}, X_h^p(t_{n+1}), H_{1h}^p(t_{n+1}), H_{2h}^p(t_{n+1}), Y_h^p(t_{n+1})\big) + \sum_{j=0}^{n} a_{j,n+1} f_3\big(t_j, X_h(t_j), H_{1h}(t_j), H_{2h}(t_j), Y_h(t_j)\big)\Big],$$

$$Y_h(t_{n+1}) = Y_0 + \frac{h^\alpha}{\Gamma(\alpha+2)}\Big[f_4\big(t_{n+1}, X_h^p(t_{n+1}), H_{1h}^p(t_{n+1}), H_{2h}^p(t_{n+1}), Y_h^p(t_{n+1})\big) + \sum_{j=0}^{n} a_{j,n+1} f_4\big(t_j, X_h(t_j), H_{1h}(t_j), H_{2h}(t_j), Y_h(t_j)\big)\Big],$$

in which

$$\begin{aligned}
X_h^p(t_{n+1}) &= X_0 + \frac{1}{\Gamma(\alpha)}\sum_{j=0}^{n} b_{j,n+1} f_1\big(t_j, X_h(t_j), H_{1h}(t_j), H_{2h}(t_j), Y_h(t_j)\big),\\
H_{1h}^p(t_{n+1}) &= H_{10} + \frac{1}{\Gamma(\alpha)}\sum_{j=0}^{n} b_{j,n+1} f_2\big(t_j, X_h(t_j), H_{1h}(t_j), H_{2h}(t_j), Y_h(t_j)\big),\\
H_{2h}^p(t_{n+1}) &= H_{20} + \frac{1}{\Gamma(\alpha)}\sum_{j=0}^{n} b_{j,n+1} f_3\big(t_j, X_h(t_j), H_{1h}(t_j), H_{2h}(t_j), Y_h(t_j)\big),\\
Y_h^p(t_{n+1}) &= Y_0 + \frac{1}{\Gamma(\alpha)}\sum_{j=0}^{n} b_{j,n+1} f_4\big(t_j, X_h(t_j), H_{1h}(t_j), H_{2h}(t_j), Y_h(t_j)\big),
\end{aligned}$$

in which the quantities

$f_1\big(t_j, X_h(t_j), H_{1h}(t_j), H_{2h}(t_j), Y_h(t_j)\big)$, $f_2\big(t_j, X_h(t_j), H_{1h}(t_j), H_{2h}(t_j), Y_h(t_j)\big)$

$f_3\big(t_j, X_h(t_j), H_{1h}(t_j), H_{2h}(t_j), Y_h(t_j)\big) and f_4\big(t_j, X_h(t_j), H_{1h}(t_j), H_{2h}(t_j), Y_h(t_j)\big)$

may be calculated by using the points $t_j = jh, j = 0, 1, \ldots, m$ as

$$\begin{aligned}
f_1(t, X(t), H_1(t), H_2(t), Y(t)) &= \lambda - \beta_1 X H_1 - \mu X + \alpha Y,\\
f_2(t, X(t), H_1(t), H_2(t), Y(t)) &= \beta_1 X H_1 - \beta_2 H_1 H_2 - (\rho + \mu) H_1,\\
f_3(t, X(t), H_1(t), H_2(t), Y(t)) &= \beta_2 H_1 H_2 - (d + \omega + \mu) H_2,\\
f_4(t, X(t), H_1(t), H_2(t), Y(t)) &= \omega H_2 - (\alpha + \gamma + \mu) Y.
\end{aligned}$$

In addition, the quantities

$$\begin{aligned}
&f_1\big(t_{n+1}, X_h^p(t_{n+1}), H_{1h}^p(t_{n+1}), H_{2h}^p(t_{n+1}), Y_h^p(t_{n+1})\big),\\
&f_2\big(t_{n+1}, X_h^p(t_{n+1}), H_{1h}^p(t_{n+1}), H_{2h}^p(t_{n+1}), Y_h^p(t_{n+1})\big),\\
&f_3\big(t_{n+1}, X_h^p(t_{n+1}), H_{1h}^p(t_{n+1}), H_{2h}^p(t_{n+1}), Y_h^p(t_{n+1})\big),
\end{aligned}$$

and

$$f_4\big(t_{n+1}, X_h^p(t_{n+1}), H_{1h}^p(t_{n+1}), H_{2h}^p(t_{n+1}), Y_h^p(t_{n+1})\big),$$

are the required estimates at $t_{n+1}, n = 0, 1, \ldots, m$.

5. Numerical and simulation results

In this section, the GABMM with initial and parameters' values provided in Table 2, [30] is used for finding numerical results of fractional order system (3). This method is a very effective tool in obtaining numerical solutions of fractional order differential equations. In interval $[0, 60]$, some graphical results are presented for the numerical solutions of system (3). The other method for the solution of system (3) which uses $\alpha = 1$, is fourth-order RKM, the corresponding computed results are compared graphically with results obtained by GABMM. The selected step size is $h = 0.0125$. Approximate solutions for $X(t), H_1(t), H_2(t), Y(t)$ and $Z(t)$ are shown in Figs. 1–5 obtained by using GABMM and the fourth-order RKM, when $\alpha = 1$ and the solutions for $X(t), H_1(t), H_2(t), Y(t)$ and $Z(t)$ are shown in Figs. 6–10, by using GABMM for the different values of α. From the graphical results in Figs. 1–5, it can be seen that the results obtained using the proposed algorithm match the results of the RK4 method very well, which implies that the presented method

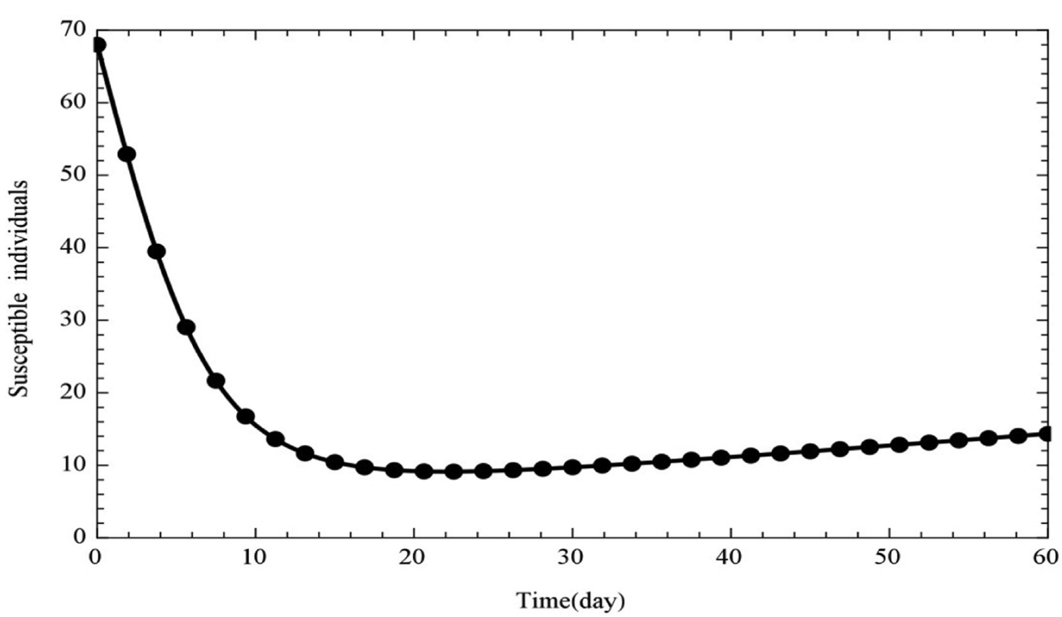

Fig. 1 $X(t)$ vs. time t: used solid line and dotted line for GABMM and RKM respectively.

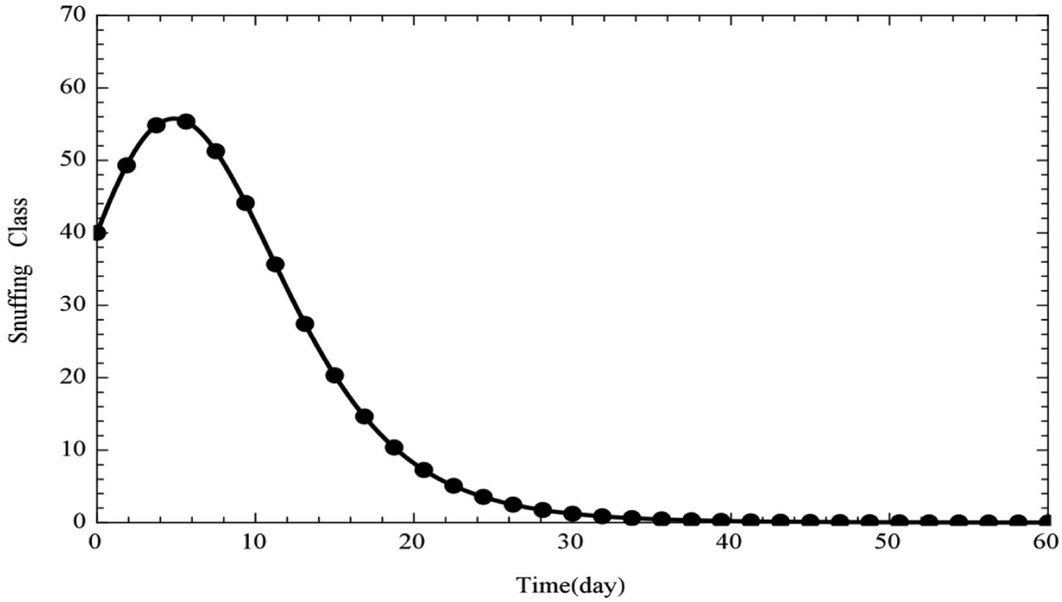

Fig. 2 $H_1(t)$ vs. time t: used solid line and dotted line for GABMM and RKM respectively.

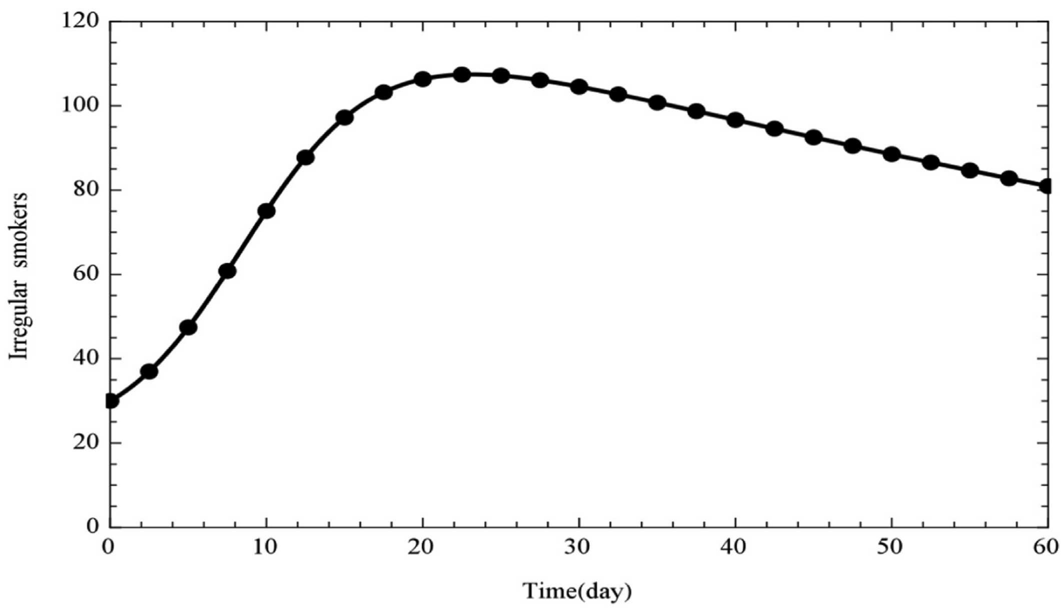

Fig. 3 $H_2(t)$ vs. time t: used solid line and dotted line for GABMM and RKM respectively.

can predict the behavior of these variables accurately in the region under consideration. Furthermore, other figures, show the approximate solutions for all considered classes obtained for different values of α using the proposed algorithm. From these graphical results, it is clear that the approximate solutions depend continuously on the time-fractional derivative.

6. Conclusions

In this manuscript, we have formulated and analyzed a new mathematical model for tobacco smoking with snuffing class. It ought to be emphasized that the model may be a generalization of a later published work proposed in [30]. Here, first, the fractional order tobacco smoking model with snuffing class is established. For a numerical solution of the proposed model, we accomplished the generalized Adams–Bashforth–Moulton method which resulted in excellent compatibility with solutions obtained with RK4 method. Also, the graphical results for the proposed model were presented. As a future work, the results could be expanded in this work to propose modern mathematical models for smoking with co-infections nature. Particularly, successful strategies to control smoking will be examined. It is noted here that analytical and numerical strategies for FDEs models arrangements are necessary.

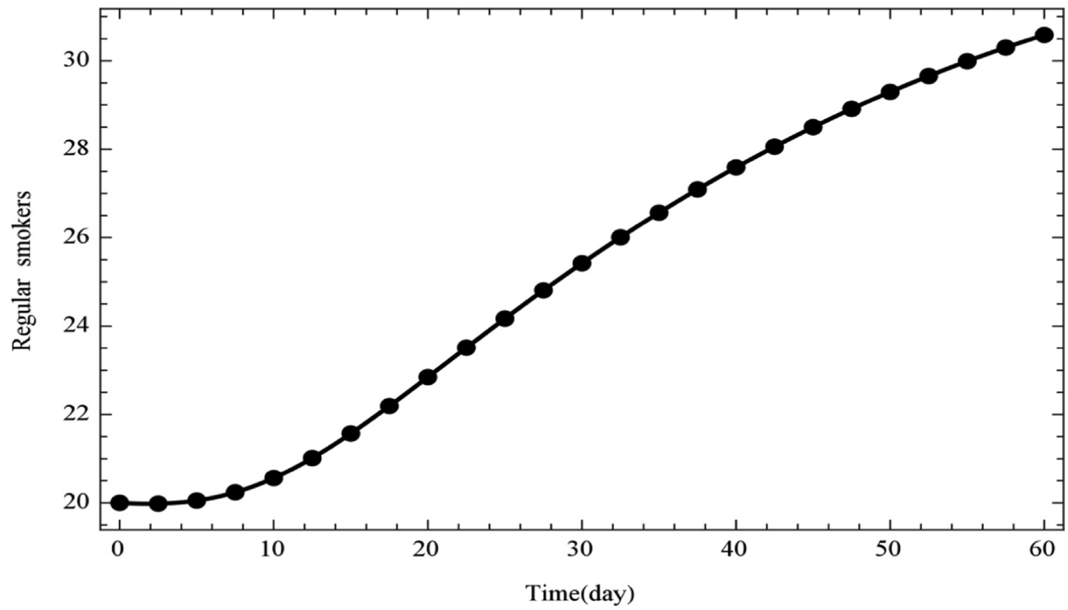

Fig. 4 $Y(t)$ vs. time t: used solid line and dotted line for GABMM and RKM respectively.

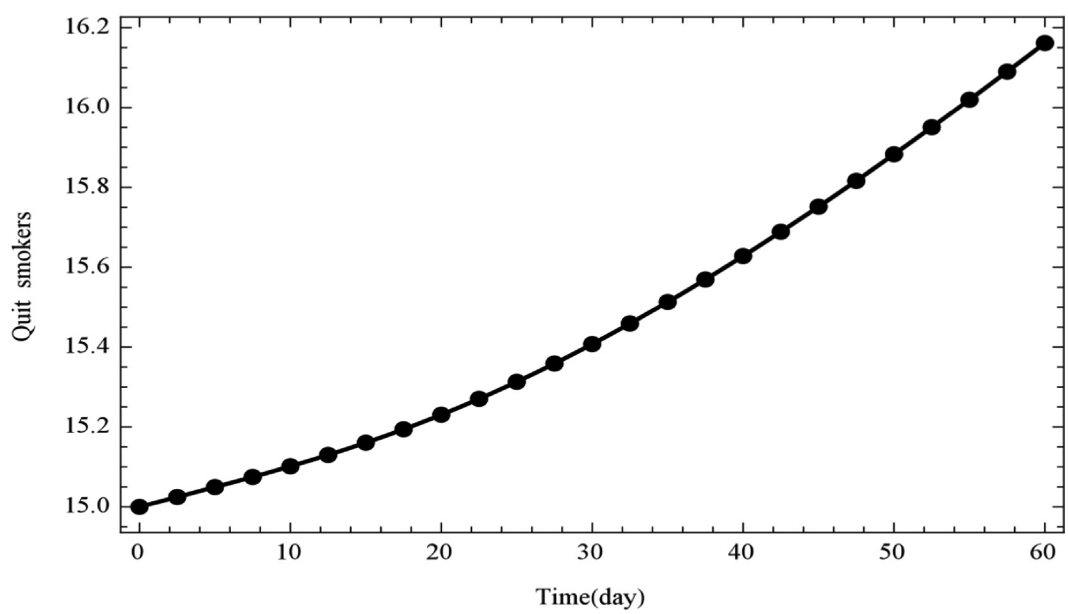

Fig. 5 $Z(t)$ vs. time t: used solid line and dotted line for GABMM and RKM respectively.

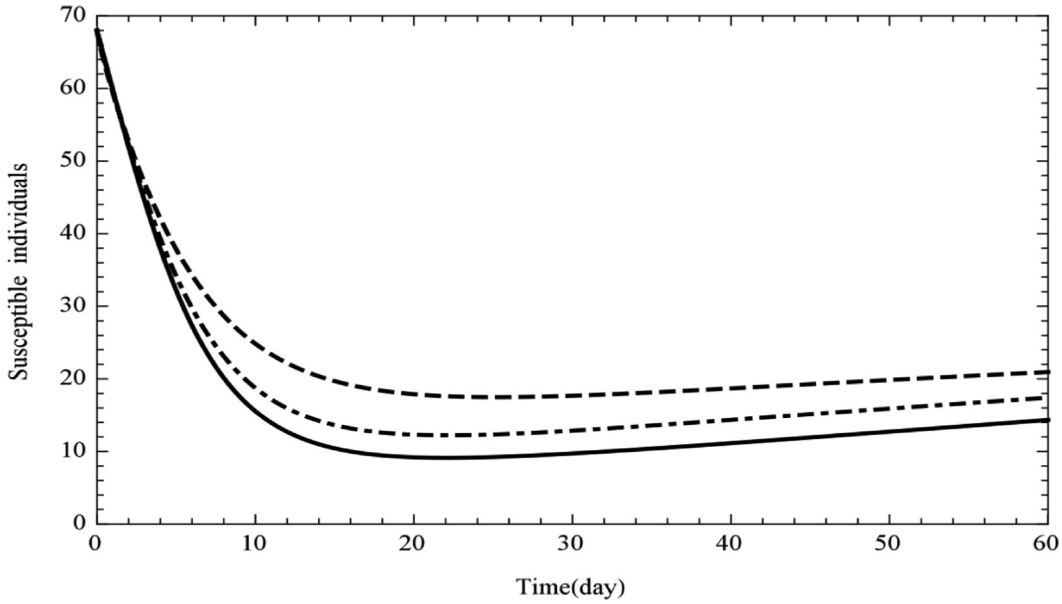

Fig. 6 $X(t)$ vs. time t: used solid line, dashed line and dot-dashed line for $\alpha = 1.0, \alpha = 0.85$ and $\alpha = 0.95$ respectively.

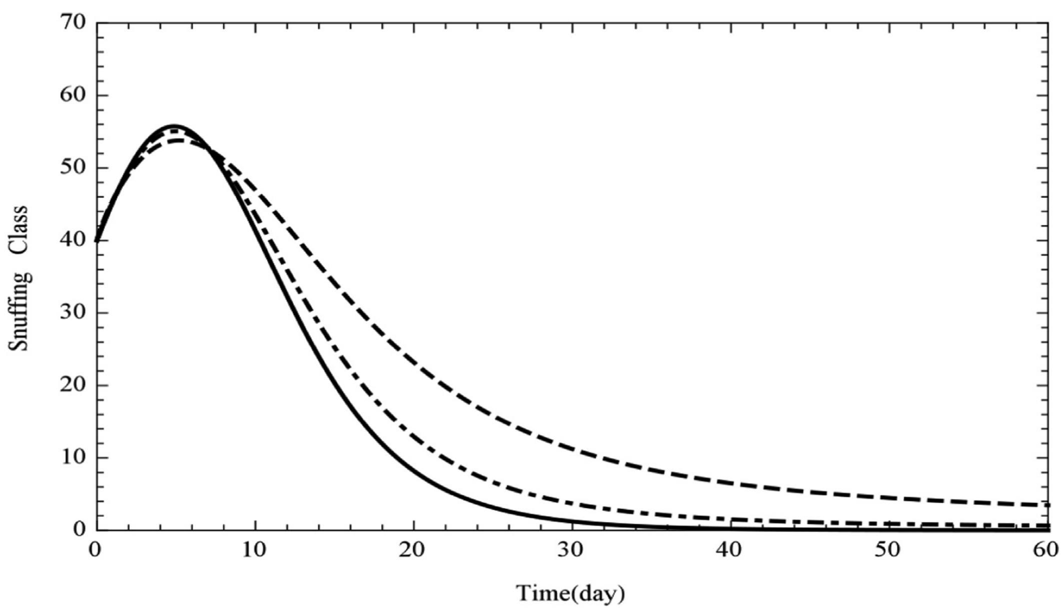

Fig. 7 $H_1(t)$ vs. time t: used solid line, dashed line and dot-dashed line for $\alpha = 1.0, \alpha = 0.85$ and $\alpha = 0.95$ respectively.

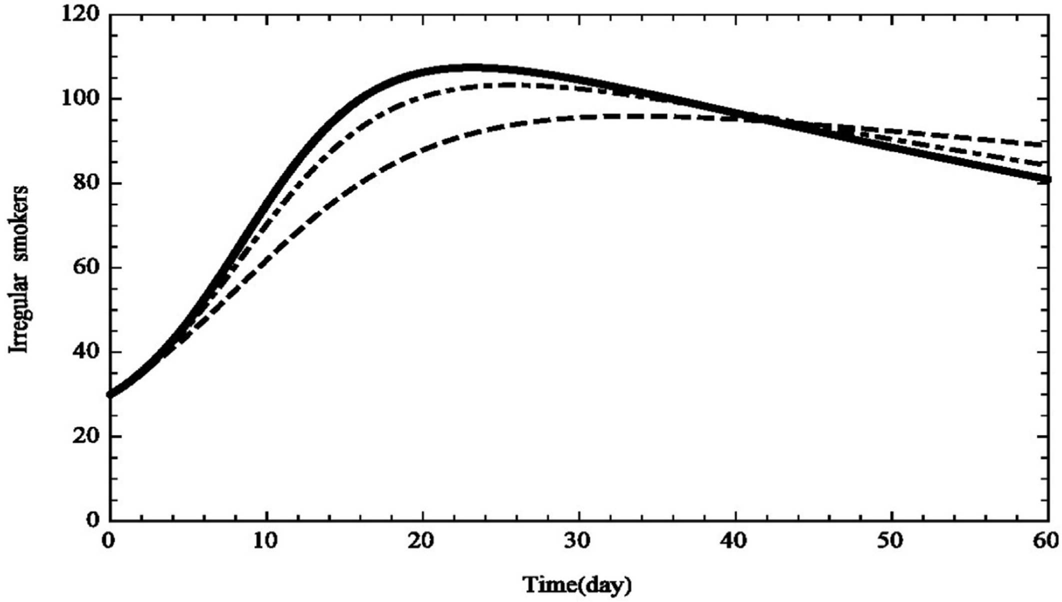

Fig. 8 $H_2(t)$ vs. time t: used solid line, dashed line and dot-dashed line for $\alpha = 1.0, \alpha = 0.85$ and $\alpha = 0.95$ respectively.

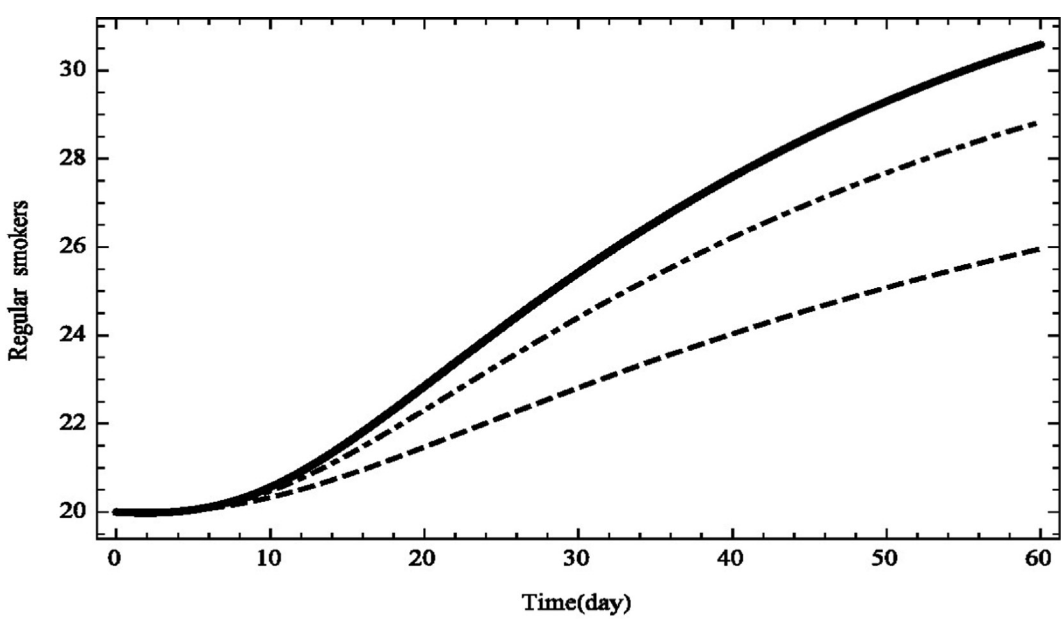

Fig. 9 $Y(t)$ vs. time t: used solid line, dashed line and dot-dashed line for $\alpha = 1.0, \alpha = 0.85$ and $\alpha = 0.95$ respectively.

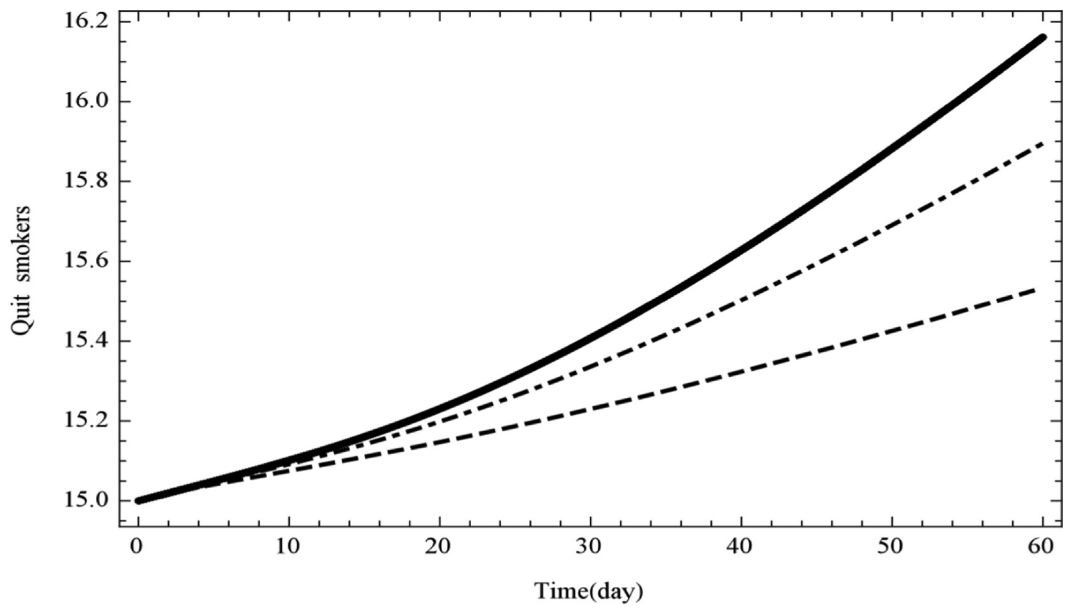

Fig. 10 $Z(t)$ vs. time t: used solid line, dashed line and dot-dashed line for $\alpha = 1.0, \alpha = 0.85$ and $\alpha = 0.95$ respectively.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

References

[1] D. Baleanu, A. Jajarmi, H. Mohammadi, S. Rezapour, A new study on the mathematical modelling of human liver with Caputo-Fabrizio fractional derivative, Chaos Solitons Fract. (2020).

[2] N.H. Tuan, H. Mohammadi, S. Rezapour, A mathematical model for COVID-19 transmission by using the Caputo fractional derivative, Chaos Solitons Fract. (2020).

[3] D. Baleanu, S. Etemad, S. Rezapour, A hybrid Caputo fractional modeling for thermostat with hybrid boundary value conditions, Bound Value Probl. 2020 (2020) 64.

[4] D. Baleanu, H. Mohammadi, S. Rezapour, A fractional differential equation model for the COVID-19 transmission by using the Caputo-Fabrizio derivative, Adv. Differ. Equ. 2020 (2020) 299.

[5] D. Baleanu, H. Mohammadi, S. Rezapour, Analysis of the model of HIV-1 infection of CD4+CD4+ T-cell with a new approach of fractional derivative, Adv. Differ. Equ. 2020 (2020) 71.

[6] S. Rezapour, H. Mohammadi, A. Jajarmi, A new mathematical model for Zika virus transmission, Adv. Differ. Equ. 2020 (2020) 589.

[7] S. Ullah, M.A. Khan, M. Farooq, A fractional model for the dynamics of TB virus, Chaos Solitons Fract. (2018).

[8] M.A. Khan, A. Atangana Modeling the dynamics of novel coronavirus (2019-nCov) with fractional derivative, Alexandr. Eng. J. (2020).

[9] S. Ullah, M.A. Khan, M. Farooq, A new fractional model for the dynamics of the hepatitis B virus using the Caputo-Fabrizio derivative, Eur. Phys. J. Plus (2018).

[10] S. Kumar, A. Kumar, B. Samet, J.F. Gómez-Aguilar, M.S. Osman, A chaos study of tumor and effector cells in fractional tumor-immune model for cancer treatment, Chaos Solitons Fract. (2021).

[11] D. Baleanu, M. Jleli, S. Kumar, B. Samet, A fractional derivative with two singular kernels and application to a heat conduction problem, Adv. Differ. Equ. (2020).

[12] I.A. Baba, A. Yusuf, K.S. Nisar, A.H. Abdel-Aty, Taher A. Nofal, Mathematical model to assess the imposition of lockdown during COVID-19 pandemic, Res. Phys. 20 (2021).

[13] S. Kumar, A. Kumar, R.P. Agarwal, B. Samet, A study of fractional Lotka-Volterra population model using Haar wavelet and Adams-Bashforth-Moulton methods, Math. Methods Appl. Sci. (2020).

[14] S. Kumar, R. Kumar, M.S. Osman, B. Samet, A wavelet based numerical scheme for fractional order SEIR epidemic of measles by using Genocchi polynomials, Numer. Methods Partial Differ. Eqs. 29 (2020).

[15] S. Kumar, S. Ghosh, R. Kumar, M. Jleli, A fractional model for population dynamics of two interacting species by using spectral and Hermite wavelets methods, Numer. Methods Partial Differ. Eqs. 29 (October 2020).

[16] S. Kumar, A. Kumar, B. Samet, H. Dutta, A study on fractional host-parasitoid population dynamical model to describe insect species, Numer. Methods Partial Differ. Eqs. 02 (2020).

[17] S. Kumar, R.P. Chauhan, S. Momani, S. Hadid, Numerical investigations on COVID?19 model through singular and non? singular fractional operators, Numer. Methods Partial Differ. Eqs. 11 (2020).

[18] C. Castillo-Garsow, G. Jordan-Salivia, A. Rodriguez Herrera, Mathematical Models for Dynamics of Tobacco Use, Recovery and Relapse, Technical Report Series BU-1505-M, Cornell Uneversity, 2000.

[19] O. Sharomi, A.B. Gumel, Curtailing smoking dynamics: A mathematical modeling approach, Appl. Math. Comput. 195 (2008) 475–499.

[20] G. Zaman, Qualitative behavior of giving up smoking models, Bull. Malays. Sci. Soc. 34 (2011) 403–415.

[21] A. Zeb, G. Zaman, S. Momani, Square root dynamic of a giving up smoking model, Appl. Math. Model. 37 (2013) 5326–5334.

[22] O.K. Ham, Stages and processes of smoking cessation among adolescents, West J. Nurs. Res. 29 (2007) 301–315.

[23] Z. Alkhudhari, S. Al- Sheikh, S. Al-Tuwarirqi, Global dynamics of mathematical model On smoking-ISRN applied mathematics, vol. 2011, pp. 7, Article ID 487075.

[24] G.A.K. van Voorn, B.W. Kooi, Smoking epidemic eradication in a eco-epidemiological dynamical model, Ecol. Complexity 14 (2013) 180–189.

[25] H.F. Huo, C.C. Zhu, Influence of relapse in a giving up smoking model, Abstr. Appl. Anal. (2013) 1–12. Article ID 525461.

[26] A. Zeb, A. Bano, E. Alzahrani, G. Zaman, Dynamical analysis of cigarette smoking model with a saturated incidence rate, AIP Adv. 8 (2018) 045317, 1–11..

[27] A. Labzai, O. Balatif, M. Rachik, Optimal control strategy for a discrete time smoking model with specific saturated incidence rate, Discrete Dyn. Nat. Soc. (2018) 1–10. Article ID 5949303.

[28] Q. Din, M. Ozair, T. Hussain, U. Saeed, Qualitative behavior of a smoking model, Adv. Differ. Eqs. (2016) 1–12. Article ID 96.

[29] Z.Z. Zhang, R.B. Wei, W.J. Xia, Dynamical analysis of a giving up smoking model with time delay, Adv. Differ. Eqs. 505 (2019) 1–17.

[30] E. Alzahrani, A. Zeb, Stability analysis and prevention strategies of tobacco smoking model, Boundary Value Problems 3 (2020) (2020) 13.

[31] G. Rahman, R.P. Agarwal, Q. Din, Mathematical analysis of giving up smoking model via harmonic mean type incidence rate, Appl. Math. Comput. 354 (2019) 128–148.

[32] A. Zeb, V.S. Erturk, U. Khan, G. Zaman, S. Momani, An approach for approximate solution of fractional-order smoking model with relapse class, Int. J. Biomath. 11 (2018) 1850077, 1–22..

[33] J. Singh, D. Kumar, M.A. Qurashi, D. Baleanu, A new fractional model for giving up smoking dynamics, Adv. Differ. Eqs. 88 (2017) 1–16.

[34] W. Lin, Global existence theory and chaos control of fractional differential equations, J. Math. Anal. Appl. 332 (1) (2007) 709–726, https://doi.org/10.1016/j.jmaa.2006.10.040.

[35] A. Gökdoğan, A. Yildirim, M. Merdan, Solving a fractional order Model of HIV Infection of CD4+ T Cells, Math. Comp. Model. 54 (9–10) (2011) 2132–2138.

[36] A.K. Alomari, A new analytic solution for fractional chaotic dynamical systems using the differential transform method, Comp. Math. Appl. 61 (9) (2011) 2528–2534, https://doi.org/10.1016/j.camwa.2011.02.043.

[37] M. Kurulay, M. Bayram, Approximate analytical solution for the fractional modified KdV by differential transform method, Com. Non. Sci. Num. Simul. 15 (7) (2010) 1777–1782, https://doi.org/10.1016/j.cnsns.2009.07.014.

[38] H. Jafari, V. Daftardar-Gejji, Solving a system of nonlinear fractional differential equations using Adomian decomposition, J. Comput. Appl. Math. 196 (2) (2006) 644–651, https://doi.org/10.1016/j.cam.2005.10.017.

[39] S. Momani, Z. Odibat, A novel method for nonlinear fractional partial differential equations: combination of DTM and generalized Taylor's formula, J. Comput. Appl. Math. 220 (2008) 85–95, https://doi.org/10.1016/j.cam.2007.07.033.

[40] Z. Odibat, S. Momani, A generalized differential transform method for linear partial differential equations of fractional order, Appl. Math. Let. 21 (2008) 194–199.

[41] V.S. Ertürk, S. Momani, Z. Odibat, Application of generalized differential transform method to multi-order fractional differential equations, Com. Non. Sci. Num. Simul. 13 (2008) 1642–1654.

[42] S. Das, Functional Fractional Calculus, Springer, 2011.

[43] R.L. Magin, Fractional Calculus in Bioengineering, Begell House Publishers, 2006.

[44] A.A. Kilbas, H.M. Srivastava, J.J. Trujillo, Theory and Applications of Fractional Differential Equations, Elsevier, Amsterdam, 2006.

[45] I. Podlubny, Fractional Differential Equations, Academic Press, New York, 1999.

[46] Z. Odibat, N. Shawagfeh, Generalized Taylor's formula, Appl. Math. Comput. 186 (1) (2007) 286–293, https://doi.org/10.1016/j.amc.2006.07.102.

[47] Z.M. Odibat, N.T. Shawagfeh, Generalized Taylor's formula, Appl. Math. Comput. 186 (1) (2007) 286–293.

[48] K. Diethelm, J. Ford, A. Freed, Detailed error analysis for a fractional Adams method, Numer. Algorithm 36 (2004) 31–52.

[49] K. Diethelm, J. Ford, A. Freed, Multi-order fractional differential equations and their numerical solution, Appl. Math. Comput. 154 (2004) 621–640.