Results in Physics 49 (2023) 106536

Contents lists available at ScienceDirect

Results in Physics

journal homepage: www.elsevier.com/locate/rinp

Stability and numerical analysis via non-standard finite difference scheme of a nonlinear classical and fractional order model

Hussam Alrabaiah [a,b], Rahim Ud Din [c,*], Khursheed J. Ansari [d], Ateeq ur Rehman Irshad [e], Burhanettin Ozdemir [e]

[a] Al Ain University, Al Ain, United Arab Emirates
[b] Mathematics Department, Tafila Technical University, Tafila, Jordan
[c] Department of Mathematics, University of Malakand, Khyber Pakhtunkhwa, Pakistan
[d] Department of Mathematics, College of Science, King Khalid University, 61413, Abha, Saudi Arabia
[e] Department of Mathematics and Sciences, Prince Sultan University, P.O. Box 66833, 11586 Riyadh, Saudi Arabia

ARTICLE INFO

Keywords:
SEABCR model
Omicron
Endemic equilibria
Reproduction number
Stability analysis
Numerical analysis and discussion

ABSTRACT

In this paper, we develop a new mathematical model for an in-depth understanding of COVID-19 (Omicron variant). The mathematical study of an omicron variant of the corona virus is discussed. In this new Omicron model, we used idea of dividing infected compartment further into more classes i.e asymptomatic, symptomatic and Omicron infected compartment. Model is asymptotically locally stable whenever $\mathcal{R}_0 < 1$ and when $\mathcal{R}_0 \leq 1$ at disease free equilibrium the system is globally asymptotically stable. Local stability is investigated with Jacobian matrix and with Lyapunov function global stability is analyzed. Moreover basic reduction number is calculated through next generation matrix and numerical analysis will be used to verify the model with real data. We consider also the this model under fractional order derivative. We use Grunwald–Letnikov concept to establish a numerical scheme. We use nonstandard finite difference (NSFD) scheme to simulate the results. Graphical presentations are given corresponding to classical and fractional order derivative. According to our graphical results for the model with numerical parameters, the population's risk of infection can be reduced by adhering to the WHO's suggestions, which include keeping social distances, wearing facemasks, washing one's hands, avoiding crowds, etc.

Introduction

The infection casus by Omicron variant of COVID-19 is called Omicron virus. In the last week of November 2021, this new invariant is identify in South Africa. After identification and repaid growth the virus transmits to other countries. As compared to other variant of corona virus omicron is not severe but transmission rate is very high. According to (CDC) Center for Diseases Control says that any one having infected by Omicron virus can transfer the infection to other vaccinated or individuals which do not have any common symptoms [1]. Some known symptoms of Omicron are runny nose, body ache, congestion, fatigue, cough etc. In order to minimize the infection rate other countries of the universe stop their flight to infected country South Africa.

The dynamics of corona virus have been studied by many scientists and researchers to control the spread of disease in the population [2–5]. To more effectively minimize the infection in population in future spreading [6–8] the scientist of the filed are tried to produce the immune and vaccinated the majority of population. Although new SARS-CoV-2 varieties have emerged with the passage of time, infections are still a problem in many nations. This paper discusses some integral order mathematical model for investigating SARS-CoV-2 infections. For instance, [9] discusses the early infection of disease in China by using a very comprehensive model. By taking into account the actual SARS-CoV-2 cases, the best elimination and control of the disease in Pakistan has been researched in [10]. The best and most efficient technique to minimize the infection is isolation and quarantine, which has explored by the authors in [11] using a mathematical modeling approach. The SARS-CoV-2 infection can spread to other uninfected persons extremely quickly. An analysis of the lockout and its effects on disease prevention using mathematical modeling can be found in [12]. The authors provided the disease control scenario for a model SEIR approaches utilizing real numerical data from France and Italy [13].

* Corresponding author.
E-mail addresses: hussam.alrabaiah@aau.ac.ae (H. Alrabaiah), rahimmaths24@gmail.com (R.U. Din), ansari.jkhursheed@gmail.com (K.J. Ansari), airshad@psu.edu.sa (A. ur Rehman Irshad), bozdemir@psu.edu.sa (B. Ozdemir).

https://doi.org/10.1016/j.rinp.2023.106536
Received 12 April 2023; Received in revised form 5 May 2023; Accepted 8 May 2023
Available online 15 May 2023

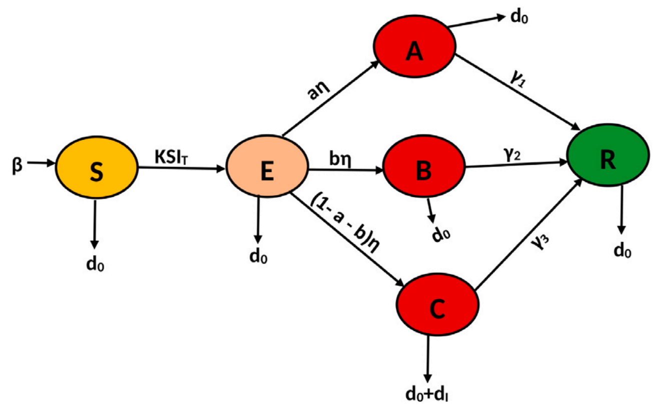

Fig. 1. Diagrammatical presentation of the model (1).

Due to its numerous attributes and practical applications to issues in engineering and physics, fractional calculus is attracting the interest of researchers all over the world. The memory, the crossover behavior, and the hereditary qualities can be shown in a fractional order model. In integral differential equations [14], the construction of the operators and applications to epidemiology [15–17], applications to follow dynamical equations [18,19], etc., the fractional calculus with various fractional operators has been found. Researchers [20] explored the SARS-CoV-2 stochastic model along time delay. Furthermore, authors [21] have used a model of fractional order to examine the singular and nonsingular kernels of the SARS-CoV-2 infection. [22] discusses the Caputo fractional model for the investigation of SARS-CoV-2 infection. The authors of [23] developed a mathematical model that takes the media campaign role into account while tackling the SARS-CoV-2 infection. [24] studies the modeling of the SAES-CoV-2 in China. The investigation and analysis of SARS-CoV-2 non reported cases are done in [25]. Authors in [26] examined how to use the new generalized idea of Caputo fractional order differentiation to arrive at the numerical analysis of the SARS-Cov-2 infecting model. [27] has examined the SARS-CoV-2 disease model with integral and non-integral orders. The time-delayed COVID-19 infection model has been proposed in [28]. Researchers [29] have looked at a fractional model to pinpoint when the illness peaked in Brazil. [30] discusses the numerical analysis of the SARS-CoV-2 model. The authors of [31] took into account the affected instances in India, created a fractional model, and then got their results. Authors [32] have discussed a SARS-CoV-2 infection model using immunization. The infection cases in Argentina have been studied using a fractional model proposed in [31]. The authors published their numerical simulations and two vaccine models for the SARS-CoV-2 infection in [32]. To analyze the infection cases in Spain, a fractional-order SARS-CoV-2 model with optimal control strategies was applied [33]. The authors examined the fractional-order SARS-CoV-2 model infection using the idea of the modified Euler approach in [34]. The conditions for the SARS-CoV-2 model's global stability in the situation of no disease were proposed by the authors [35]. A nonlinear fractional order SARS-CoV-2 model infection has been proposed in [36]. To explore the qualitative analysis of the model, a mathematical model for SARS-CoV-2 has been built in [37]. Moreover authors [38] have investigated a model on the SARS-CoV-2 for global dynamics. In additions, the concepts of fractional calculus have been used very well recently to investigate COVID-19. Additionally, various models involving concepts of mathematical analysis have been utilized very well. In this regards we refer some work, where various analysis and numerical scheme have been used to study different kinds of problems as [39–50].

The primary objective of this paper is to use mathematical modeling to comprehend the behavior of the new SARS-CoV-2 type known as the Omicron. We develop the model using the omicron feature and parameterize it using real data from South Africa. In order to explore the potential existence of several layers or waves, a NCOVID-19 mathematical model for new variant (omicron) is taken into consideration. Sensitivity analysis is used to identify the variables that have the greatest potential to improve or reduce the fundamental reproduction number R_0.

This study present epidemics model for SARS-CoV-2 regarding focus on strategies, control and specific vaccination of population. Proposed model are solve for Reproduction Number, analysis and model description respectively. Numerical simulation are calculated by NSFD Scheme [51]. A general group of techniques in numerical analysis known as non-finite difference schemes create a discrete model to provide numerical solutions to differential equations. In the last conclude this paper by 'Conclusion'.

Model formulation

Due mainly to the new SARS-CoV-2 version, also called Omicron virus, which was first discovered in South Africa, people there once more had to adhere to tight SOPs, keep social distances, wear face masks, and other limitations. Because of the new SARS-CoV-2 variant's rapid proliferation in several countries, these nations prohibited their citizens from traveling to South Africa. We are developed a new model to comprehend the dynamical behavior of the COVID-19 cases from South Africa while keeping in mind the Omicron variation. We take into account the entire population, indicating it by $\mathcal{N}(t)$, and $\mathcal{I}$ is divided in $\mathcal{I} = \mathcal{A} + \mathcal{B} + \mathcal{C}$. Further classifying it into six distinct epidemiological groups: Those who have been exposed to $\mathcal{S}(t)$, represent the uncontagious population, and $\mathcal{I}\mathcal{E}(t)$ is stand for exposed population, which coming into touch with asymptomatic population, symptomatic population, or omicron variant-infected individuals; asymptomatic population are represent $\mathcal{A}(t)$; and symptomatic population by $B(t)$ (infected individuals population although they have no clear symptoms sign), symptomatic people (population who exhibit clinical symptoms consistent with SARS-CoV-2 infections), having Omicron variant infection is represent by $C(t)$ (who exhibit clear clinical symptom consistent with omicron infection; and have the ability to spread the disease to others whether or not they have received the vaccine), the recovered population is represent by, $\mathcal{R}(t)$ (population recovered from $\mathcal{A}(t)$, $\mathcal{B}(t)$ and $C(t)$). We obtain our model as

$$\begin{aligned}
\frac{d\mathcal{S}}{dt} &= \beta - k\mathcal{S}\mathcal{I} - d_0\mathcal{S},\\
\frac{d\mathcal{E}}{dt} &= k\mathcal{S}\mathcal{I} - (\eta + d_0)\mathcal{E},\\
\frac{d\mathcal{A}}{dt} &= a\eta\mathcal{E} - (\gamma_1 + d_0)\mathcal{A},\\
\frac{d\mathcal{B}}{dt} &= (1 - a - b)\eta\mathcal{E} - (\gamma_2 + d_0 + d_I)\mathcal{B},\\
\frac{d\mathcal{C}}{dt} &= b\eta\mathcal{E} - (\gamma_3 + d_0)\mathcal{C},\\
\frac{d\mathcal{R}}{dt} &= \gamma_1\mathcal{A} + \gamma_2\mathcal{B} + \gamma_3\mathcal{C} - d_0\mathcal{R}.
\end{aligned} \tag{1}$$

The flow of infection and different rate from one compartment to another is presented as in Fig. 1.

Symbols involved in the model (1) are described in Table 1.

Feasible region, positivity and boundedness

On addition of all equations of the model (1) yields

$$\mathcal{N}(t) = \mathcal{S}(t) + \mathcal{E}(t) + \mathcal{A}(t) + \mathcal{B}(t) + \mathcal{C}(t) + \mathcal{R}(t),$$

one has

$$\begin{aligned}
\frac{d\mathcal{N}(t)}{dt} &= \beta - d_0\mathcal{N} - d_I\mathcal{B}. \\
&\le \beta - d_0\mathcal{N}
\end{aligned} \tag{2}$$

Table 1
Nomenclature and discerption of the model (1).

Variables	The physical representation
$\mathcal{S}$	Susceptible class
$\mathcal{E}$	Exposed class
$\mathcal{A}$	Asymptomatic compartment
$\mathcal{B}$	Symptomatic with no visible symptoms compartment
$\mathcal{C}$	Omicron infected compartment
$\mathcal{R}$	Recovered compartment
β	New emergent population
d_0	Natural death rate
γ_1	Recovery from Asymptomatic
γ_2	Recovery from Symptomatic
γ_3	Recovery from omicron infected
d_I	Death rate from Symptomatic
a	Infection rate of asymptomatic
η	Exposed rate
b	Infection rate of symptomatic
k	Contact rate

Table 2
The parameters and their discerption involve in the model (1).

Nomenclature	Numerical values
$\mathcal{S}_0$	6.0069540 millions
$\mathcal{E}_0$	0.062000 millions
$\mathcal{A}_0$	0.008000 millions
$\mathcal{B}_0$	0.000100 millions
$\mathcal{C}_0$	0..000360 millions
$\mathcal{R}_0$	0 millions
β	0.2553
d_0	0.00425
γ_1	0.8447
γ_2	0.200
γ_3	0.6746
d_I	0.0015
b	0.0101
η	0.8999
a	0.9566
k	0.8999

From (2), we have

$$\mathcal{N}(t) = \frac{\beta}{d_0} + \left(\mathcal{N}_0 - \frac{\beta}{d_0}\right) e^{d_0 t}. \tag{3}$$

All values of non-negative for t.

Therefore, all solutions began of the system (1) will remain nonnegative for all value of t is equal to zero. So, the system (1) is well-posed mathematically.

The feasible region for the dynamical analysis is following

$$\chi = (\mathcal{S}, \mathcal{E}, \mathcal{A}, \mathcal{B}, \mathcal{C}, \mathcal{R}) : 0 \le \mathcal{S}, \mathcal{S} + \mathcal{E} + \mathcal{A} + \mathcal{B} + \mathcal{C} + \mathcal{R} \le \frac{\beta}{d_0}.$$

Disease-free Equilibrium (DFE)

The DFE of the system (1) is denoted by $\mathcal{E}_0 = (\mathcal{S}^0, 0, 0, 0, 0, 0)$ which is follow

$$\mathcal{E}^0 = \left(\frac{\beta}{d_0}, 0, 0, 0, 0, 0\right). \tag{4}$$

Endemic equilibrium (EE)

Also the EE is computed as

$$\mathcal{S}^*(t) = \frac{\beta}{k\mathcal{I}^*},$$
$$\mathcal{E}^*(t) = \frac{k\beta\mathcal{I}^*}{(k\mathcal{I}^* + d_0)(\eta + d_0)},$$
$$\mathcal{A}^*(t) = \frac{ka\eta\beta\mathcal{I}^*}{(k\mathcal{I}^* + d_0)(\eta + d_0)(\gamma_1 + d_0)},$$
$$\mathcal{B}^*(t) = \frac{(1 - a - b)k\beta\mathcal{I}^*}{(k\mathcal{I}^* + d_0)(\eta + d_0)(\gamma_1 + d_0)(\gamma_2 + d_0 + d_I)},$$
$$\mathcal{C}^*(t) = \frac{kb\eta\beta\mathcal{I}^*}{(k\mathcal{I}^* + d_0)(\eta + d_0)(\gamma_1 + d_0)(\gamma_3 + d_0)},$$
$$\mathcal{R}^*(t) = \frac{\gamma_1\mathcal{A}^* + \gamma_2\mathcal{B}^*\gamma_3\mathcal{C}^*}{d_0}.$$

Expression for $\mathscr{R}_0$

In epidemiology there is a factor, which called the basic reproduction number $\mathscr{R}_0$ which describe the control and transmission of infection. Furthermore from $\mathscr{R}_0$, we can determine how the infection of COVID-19 is going in the population and which is the greatest choice to control the infection from the population. The next generation matrix method is used to find $\mathscr{R}_0$ is given below let $\chi = (\mathcal{E}, \mathcal{A}, \mathcal{B}, \mathcal{C})$, then form system (1), we have

$$\frac{d\chi}{dt} = \mathcal{F} - \mathcal{V}.$$

Where

$$\mathcal{F} = \begin{pmatrix} k\mathcal{S}\mathcal{I} \\ 0 \\ 0 \\ 0 \end{pmatrix}.$$

And

$$\mathcal{V} = \begin{pmatrix} -(\eta + d_0)\mathcal{E} \\ -a\eta\mathcal{E} + (\gamma_1 + d_0)\mathcal{A} \\ -(1 - a - b)\eta\mathcal{E} - (\gamma_2 + d_0 + d_I)\mathcal{B} \\ b\eta\mathcal{E} - (\gamma_3 + d_0)\mathcal{C} \end{pmatrix}.$$

Jacobian of $\mathcal{F}$ at DFE point is

$$F = \begin{pmatrix} 0 & k\mathcal{S}^0 & k\mathcal{S}^0 & k\mathcal{S}^0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}.$$

Also for the DFE, Jacobian of matrix $\mathcal{V}$ is follow

$$V = \begin{pmatrix} \eta + d_0 & 0 & 0 & 0 \\ -a\eta & \gamma_1 + d_0 & 0 & 0 \\ -(1 - a - b)\eta & 0 & \gamma_2 + d_0 + d_I & 0 \\ -b\eta & 0 & 0 & \gamma_3 d_0 \end{pmatrix}.$$

Hence, $\mathcal{R}_0$ is spectral radius of FV^{-1} computed as

$$\mathcal{R}_0 = \frac{kb\eta}{(\gamma_3 + d_0)(\eta + d_0)} + \frac{ka\eta}{(\gamma_1 + d_0)(\eta + d_0)} + \frac{k(1 - a - b)\eta}{(\gamma_2 + d_0 + d_I)(\eta + d_0)}.$$

Hence, $\mathcal{R}_0$ has three parts, first part represent contact between infected population with uninfected (healthy) population. While second part represent the contact between uninfected population to asymptomatic and third part show infection of symptomatic population. The infection of SARS-CoV-2 can be control in the population whenever the valve of $\mathcal{R}_0 < 1$, on the other hand the infection will be spread in the community if $\mathcal{R}_0 > 1$. In next theorem, we discus the stability of the system (1) under condition of $\mathcal{R}_0 > 1$ and $\mathcal{R}_0 < 1$ at disease-free equilibrium. Using the numerical value of Table 2, we give a 3D profile of $\mathcal{R}_0$ in Fig. 2.

Theorem 1. *"(i) There does not exits positive equilibrium for the model (1), if $\mathscr{R}_1 \le 1$ or/and $\mathscr{R}_2 \le 1$.*

(ii) There exists a distinct positive (unique) equilibrium $\mathcal{E}^* = (\mathcal{S}^*, \mathcal{E}^*, \mathcal{A}^*, \mathcal{B}^*, \mathcal{C}^*, \mathcal{R}^*)$ which is also called an endemic equilibrium if $\mathscr{R}_1 > 1$ or/and $\mathscr{R}_2 > 1$."

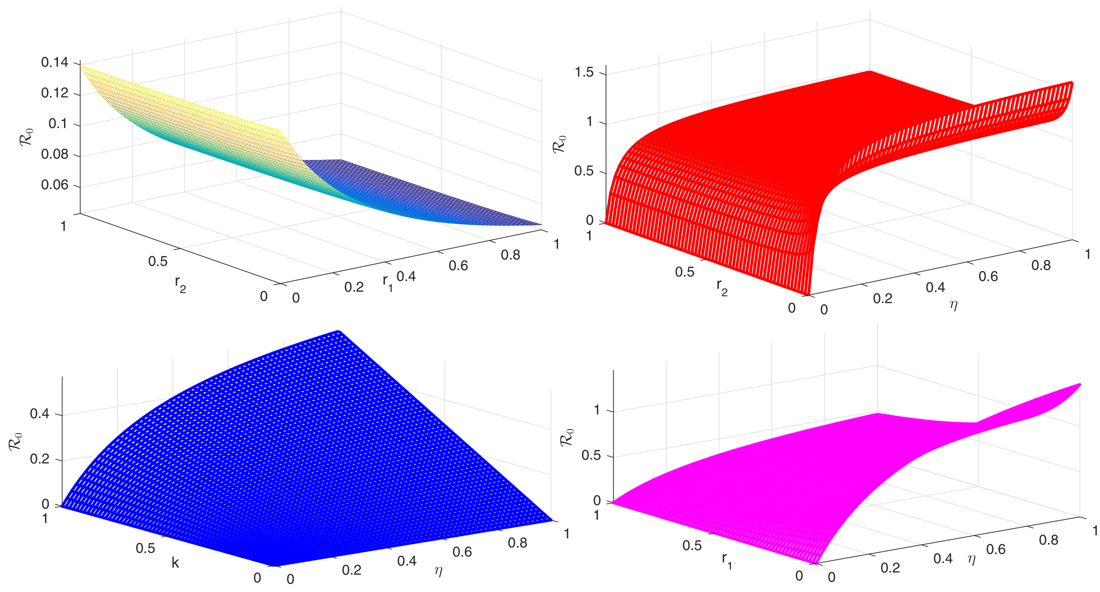

Fig. 2. 3D profile of $\mathcal{R}_0$.

Stability analysis

This section has the study of properties of equilibrium point with local and global stability of the system (1). Also, we will investigate the properties of these equilibrium and global analysis of the system (1).

Local stability

In this section, we present the local stability of the model (1) with condition $\mathcal{R}_0 < 1$.

Theorem 2. *"If $\mathscr{R}_0 < 1$, infection model COVID-19 is stable locally asymptotically at DFE $\mathcal{E}_0$."*

Proof. To obtained the stability result, the Jacobian Matrix at $\mathcal{E}_0$ is given below

$$\mathcal{M} = \begin{bmatrix} -d_0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -(d_0+\eta) & 0 & 0 & 0 & 0 \\ 0 & a\eta & -(\gamma_1+d_0) & 0 & 0 & 0 \\ 0 & (1-a-b)\eta & 0 & -(\gamma_2+d_0+d_I) & 0 & 0 \\ 0 & b\eta & 0 & 0 & -(\gamma_3+d_0) & 0 \\ 0 & 0 & \gamma_1 & \gamma_2 & \gamma_3 & -d_0 \end{bmatrix}$$

The characteristic polynomial of the Jacobian matrix at DFE is given by $det(\mathcal{M}^0 - \lambda I) = 0$, where λ is the eigenvalue and I is 6 by 6 identity matrix. Thus, $\mathcal{M}$ has eigenvalues given by

$\lambda_1 = -d_0$,

$\lambda_2 = -(\eta + d_0)$,

$\lambda_3 = -(\gamma_1 + d_0)$,

$\lambda_4 = -(\gamma_2 + d_0 + d_I)$,

$\lambda_5 = -(\gamma_3 + d_0)$,

$\lambda_6 = -d_0$.

λ_1, λ_2, λ_3, λ_4, λ_5 and λ_6 are negative strictly. Thus, Disease free stability (DEF) is stable locally asymptotically. □

Global stability

In this part of our manuscript, we present global stability at DFE of the system (1). We constructed a function called Lyapunov function to show the model (1) is stable asymptotically globally in the next theorem.

Theorem 3. *" If $\mathcal{R}_0 \leq 0$, then infection of system (1) is stable globally asymptotically at $\mathcal{E}_0$."*

Proof. To prove this theorem result, we constructed a function called Lyapunov function.

$$\mathcal{W} = C_1(\mathcal{S} - \mathcal{S}^0) + C_2(\mathcal{E} - \mathcal{E}^0) + C_3(\mathcal{A} - \mathcal{A}^0) + C_4(\mathcal{B} - \mathcal{B}^0) + C_5\mathcal{C}. \quad (5)$$

Here C_1, C_2, C_3 and C_4 are some constants. Now w.r.t "t" take derivative of above equation.

$$\dot{\mathcal{W}} = C_1\dot{\mathcal{S}} + C_2\dot{\mathcal{E}} + C_3\dot{\mathcal{A}} + C_4\dot{\mathcal{B}} + C_5\dot{\mathcal{C}}.$$

Putting the values from (1)

$$\dot{\mathcal{W}} = C_1\beta - k\mathcal{S}\dot{\mathcal{I}} - d_0\mathcal{S} + C_2k\mathcal{S}\mathcal{I} - (\dot{\eta} + d_0)\mathcal{E} + C_3a\eta\mathcal{E} + (\dot{\gamma_1} + d_0)\mathcal{A}$$
$$+ C_4(1-a-b)\eta\mathcal{E} + \dot{(}\gamma_2 + d_0 + d_I)\mathcal{B} + C_5b\eta\mathcal{E} + (\gamma_3 + d_0)\mathcal{C}.$$

After some basic calculation

$$\dot{\mathcal{W}} = k\mathcal{S}\mathcal{I}(C_2 - C_1) + a\eta\mathcal{E}(C_3 - C_4) + \eta\mathcal{E}(C_4 - C_2) + b\eta\mathcal{E}(C_5 - C_4)$$
$$-(C_1d_0\mathcal{S} - C_1\beta) - C_2d_0\mathcal{E} - C_3\gamma_1\mathcal{A} - c_3d_0\mathcal{A} - C_4\gamma_2\mathcal{B}$$
$$-C_4d_0\mathcal{B} - C_4d_I\mathcal{B} - C_5\gamma_3\mathcal{C} - C_5d_0\mathcal{C}.$$

Let us choose $C_1 = C_2 = C_3 = C_4 = C_5 1$.

We obtained

$$\dot{\mathcal{W}} = -(d_0\mathcal{S} - \beta) - d_0\mathcal{E} - \gamma_1\mathcal{A} - d_0\mathcal{A} - \gamma_2\mathcal{B} - d_I\mathcal{B} - \gamma_3\mathcal{C} - d_0\mathcal{C} < 0$$

Hence, $\mathcal{R}_0 \leq 0$ then the system (1) is globally asymptotically stable. □

Numerical results and discussion

We take into account the infected cases that were recorded in South Africa. The infection are evaluated on a regular basis using a unit per day. Calculated natural birth rate and death rate 0.0155432, which is the mean rate of life of South Africans in 2020–21, are two of the 12 parameters in the given model (1). The method $N(0) = \frac{\beta}{d_0}$ is used to calculate the birth rate, here $N(0)$ represent total population of South Africa in 2021. About $N(0) = 60140000$ people are projected to live in South Africa in 2021. $\mathcal{S}(0) = 60069540$ is the uninfected population individual in the non-appearance of the disease, and $\mathcal{E}(0) = 0.062$

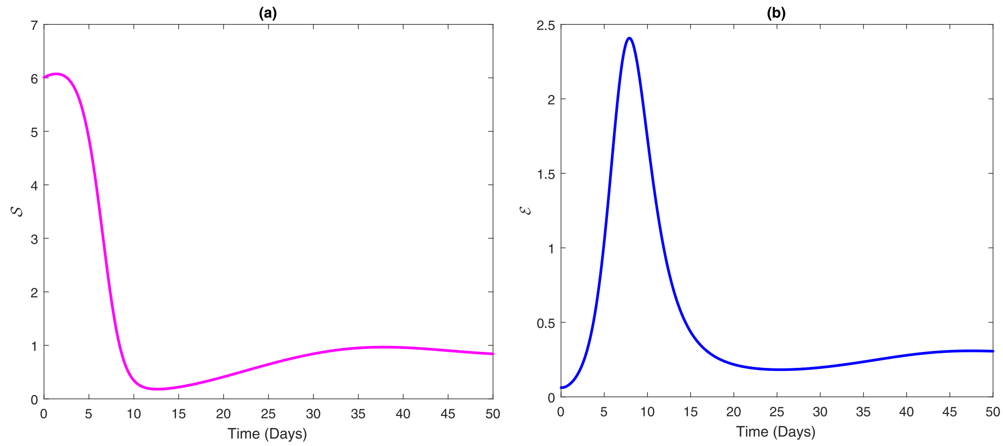

Fig. 3. Dynamical behaviors of susceptible and exposed classes of the considered model corresponding to classical model.

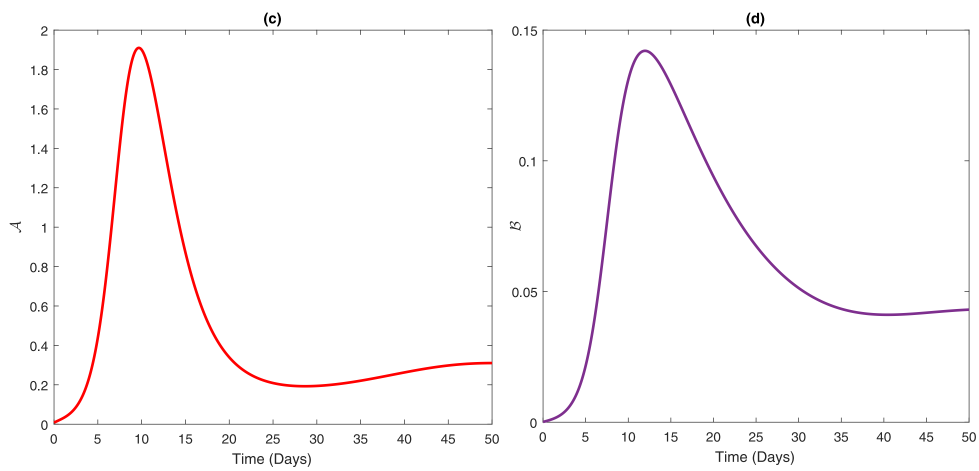

Fig. 4. Dynamical behaviors of symptomatic and symptomatic with no visible symptoms classes of the considered model corresponding to classical model.

million, $\mathcal{A}(0) = 0.008$ million, $\mathcal{B}(0) = 100$, and $\mathcal{C}(0) = 360$ are the reported people on 1st November, 2021, with $\mathcal{R}(0) = 0$ being the initial given conditions susceptible to fitting data. Let suppose that recovery from infection is zero yet. In order to suitable the system to the indicated numerical data with the time mentioned early, we used the nonlinear least square method. The experiments were carried out up until the required level of accuracy in the model fitting. Here, we establish NSFD scheme [51] for our considered model. Therefore, consider first equation of (1) as

$$\frac{d\mathcal{S}(t)}{dt} = \beta - k\mathcal{S}\mathcal{I} - d_0\mathcal{S}. \tag{6}$$

Which is decomposed by NSFD scheme as

$$\frac{\mathcal{S}_{j+1} - \mathcal{S}_j}{h} = \beta - k\mathcal{S}_j\mathcal{I}_j - d_0\mathcal{S}_j. \tag{7}$$

Like (7), by utilizing NSFD method, we present the model (1) as

$$\mathcal{S}_{j+1} = \mathcal{S}_j + h\left(\beta - k\mathcal{S}_j\mathcal{I}_j - d_0\mathcal{S}_j\right),$$

$$\mathcal{E}_{j+1} = \mathcal{E}_j + h\left(k\mathcal{S}_j\mathcal{I}_j - (\eta + d_0)\mathcal{E}_j\right), \tag{8}$$

$$\mathcal{A}_{j+1} = \mathcal{I}_j + h\left(a\eta\mathcal{E}_j - (\gamma_1 + d_0)\mathcal{A}_j\right) \tag{9}$$

$$\mathcal{B}_{j+1} = \mathcal{V}_j + h\left((1 - a - b)\eta\mathcal{E}_j - (\gamma_2 + d_0 + d_I)\mathcal{B}_j\right), \tag{10}$$

$$\mathcal{C}_{j+1} = \mathcal{R}_j + h\left(b\eta\mathcal{E}_j - (\gamma_3 + d_0)\mathcal{C}_j\right),$$

$$\mathcal{R}_{j+1} = \mathcal{R}_j + h\left(\gamma_1\mathcal{A}_j + \gamma_2\mathcal{B}_j + \gamma_3\mathcal{C}_j - d_0\mathcal{R}_j\right).$$

Developing NSFD scheme [51], we plotted the model (1). To understand the dynamics of the system, we used real data of South Africa to simulate. With using numerical value table, we present graphically the dynamical behaviors of various compartments of our model in Figs. 3–5 using classical order model.

NSFD Scheme for Fractional Order Model (1)

In this part of our paper, we present some fractional concept for model (1), we used Grunwald–Letnikov method for Caputo derivative as used in [41,42]. Moreover, the numerical method used in this paper has the ability to demonstrate the dynamical properties of model (1). Taking a small interval h, our using numerical method preserve the stability of disease-free and endemic equilibrium points. Mostly researchers used different methods like Euler, RK2, and RK4 method but our numerical method have the advantages over it. For instance, the said numerical scheme has less complexity than RK2 and RK4 methods. This numerical scheme has a significant advantage over other scheme that for a large time raising it yield a good numerical results.

Definition 4. "Integral of non-integer order $\beta > 0$ of a function $\mathcal{S} : [0, \infty) \to \mathbb{R}$ is defined as

$$I_t^{\varphi}\mathcal{X}(t) = \frac{1}{\chi(\alpha)} \int_0^t \frac{\mathcal{X}(\theta)}{(t - \theta)^{1-\alpha}} d\theta,$$

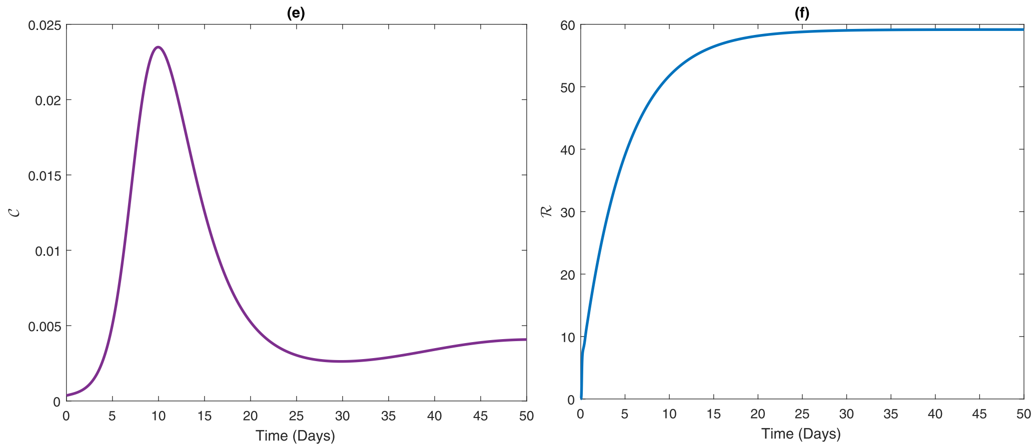

Fig. 5. Dynamical behaviors of omicron infected and recovered classes of the considered model corresponding to classical model.

provided the integral exists at the right sides". Moreover, by definition of Caputo, we have

$$D^{\alpha}_{0+}\mathcal{X}(t) = \begin{cases} \dfrac{1}{\chi(1-\alpha)}\displaystyle\int_0^t (t-\eta)^{-\alpha}\mathcal{X}'(\eta)d\eta, \ 0<\alpha\le 1, \\ \dfrac{d\mathcal{X}}{dt}, \quad \alpha = 1. \end{cases}$$

From Riemann Liouville operator integral with some fractional order as

$$I^{\alpha}\chi(t) = \frac{1}{\Gamma(\alpha)}\int_0^t (t-\tau)^{\alpha-1}\chi(\theta)d\theta, \ \alpha>0. \quad (11)$$

Following the procedure of [42], we have

$$^{C}\mathrm{D}^{\alpha}\chi(t) = \sum_{j=0}^{n+1} N_j^{\alpha}\chi_{n-j+1}, \ n = 0,1,2,\dots, \quad (12)$$

where $N_0 = \frac{1}{\mathfrak{h}^{\alpha}}$, $N_j = \left(\frac{j-1-\alpha}{j}\right)N^{\alpha}_{j-1}$, $j = 1,2,3,\dots$. Here it is interesting if $n = 0$, we have from (12)

$$^{C}\mathrm{D}^{\alpha}\chi(t) = \frac{\chi_1 - \alpha\chi_0}{\mathfrak{h}^{\alpha}}.$$

Consider

$$\begin{cases} ^{C}\mathrm{D}^{\alpha}\chi(t) = f(t,\chi(t)), \ t\in[0,T], 0<T<\infty, \\ \chi(t_0) = \chi_0. \end{cases} \quad (13)$$

From Eq. (12) to discretion (13) as

$$\sum_{j=0}^{n+1} N_j^{\alpha}\chi_{n-j+1} = f(t_{n+1},\chi(t_{n+1})), \ n = 0,1,2,\dots, \quad (14)$$

we have

$$\chi_{n+1} = \frac{1}{N_0^{\alpha}}\left[-\sum_{j=1}^{n+1} N_j^{\alpha}\chi_{n-j+1} + f(t_{n+1},\chi(t_{n+1}))\right], \text{ where } n = 0,1,2,\dots, \quad (15)$$

where $N_0^{\alpha} = \left[\frac{1}{\varphi(\mathfrak{h},\omega)}\right]^{\alpha}$, $N_j^{\alpha} = \left[\frac{j-1-\alpha}{j}\right]N^{\alpha}_{j-1}$, $j = 1,2,3,\dots$. Let assume $\varphi(\mathfrak{h},\omega)$ obtained $\mathfrak{h} + O(\mathfrak{h}^2)$. The stability rule is obtained from some functions as $\sin\mathfrak{h}, \cos\mathfrak{h}, \mathfrak{h}$, etc. Here, we confer our counsel system (1) to non-integral order $0<\alpha\le 1$ as follow

$$\begin{cases} ^{C}\mathrm{D}^{\alpha}S(t) = \beta - k\mathcal{S}\mathcal{I} - d_0\mathcal{S}, \\ ^{C}\mathrm{D}^{\alpha}E(t) = k\mathcal{S}\mathcal{I} - (\eta + d_0)\mathcal{E}, \\ ^{C}\mathrm{D}^{\alpha}A(t) = a\eta\mathcal{E} - (\gamma_1 + d_0)\mathcal{A}, \\ ^{C}\mathrm{D}^{\alpha}B(t) = (1-a-b)\eta\mathcal{E} - (\gamma_2 + d_0 + d_I)\mathcal{B}, \\ ^{C}\mathrm{D}^{\alpha}C(t) = b\eta\mathcal{E} - (\gamma_3 + d_0)\mathcal{C}, \\ ^{C}\mathrm{D}^{\alpha}R(t) = \gamma_1\mathcal{A} + \gamma_2\mathcal{B} + \gamma_3\mathcal{C} - d_0\mathcal{R}. \end{cases} \quad (16)$$

Considering of (15) and by utilizing Grunwald–Letnikov discrimination method, for system (16) for non-integral order, we present numerical method here.

$$\begin{cases} S(t_{n+1}) = \dfrac{1}{N_0^{\alpha}}\left[-\displaystyle\sum_{j=1}^{n+1} N_j^{\alpha}S(t_{n+1-j}) + \beta - k\mathcal{S}(t_n)\mathcal{I}(t_n) - d_0\mathcal{S}(t_n)\right], \\ E(t_{n+1}) = \dfrac{1}{N_0^{\alpha}}\left[-\displaystyle\sum_{j=1}^{n+1} N_j^{\alpha}E(t_{n+1-i}) + k\mathcal{S}(t_n)\mathcal{I}(t_n) - (\eta + d_0)\mathcal{E}(t_n)\right], \\ A(t_{n+1}) = \dfrac{1}{N_0^{\alpha}}\left[-\displaystyle\sum_{j=1}^{n+1} N_j^{\alpha}A(t_{n+1-i}) + a\eta\mathcal{E}(t_n) + (\gamma_1 + d_0)\mathcal{A}(t_n)\right], \\ B(t_{n+1}) = \dfrac{1}{N_0^{\alpha}}\left[-\displaystyle\sum_{j=1}^{n+1} N_j^{\alpha}B(t_{n+1-i}) + (1-a-b)\eta\mathcal{E}(t_n) + (\gamma_2 + d_0 + d_I)\mathcal{B}(t_n)\right], \\ C(t_{n+1}) = \dfrac{1}{N_0^{\alpha}}\left[-\displaystyle\sum_{j=1}^{n+1} N_j^{\alpha}C(t_{n+1-i}) + b\eta\mathcal{E}(t_n) + (\gamma_3 + d_0)\mathcal{C}(t_n)\right] \\ R(t_{n+1}) = \dfrac{1}{N_0^{\alpha}}\left[-\displaystyle\sum_{j=1}^{n+1} N_j^{\alpha}R(t_{n+1-i}) + \gamma_1\mathcal{A}(t_n) + \gamma_2\mathcal{B}(t_n) + \gamma_3\mathcal{C}(t_n) - d_0\mathcal{R}(t_n)\right]. \end{cases} \quad (17)$$

Here in Figs. 6–8, we present graphically the dynamical behaviors of various classes using different fractional orders.

Conclusion

In this manuscript, we constructed a new mathematical SEIVR model for NCOVID-19, which is base on behavior of virus. Our main goal is to estimate the effect of immunization on population. In the presence of presenting no clear infection or symptoms and loss of immunity are the root measure which differentiate this virus from other modeled infectious diseases. On the bases of analytical expression for $\mathcal{R}_0$ which is a main element to determine necessary and sufficient conditions for disease-free and endemic equilibrium. We testify our theoretical results with real data for a case of South Africa.

Our main resulting and finding are follow.

Vaccination helps to overcome the disease, we provide a mathematical proof that transmission rates are reduce with vaccination. We mathematical justified that the increasing in number of infection in South Africa was followed by vaccination campaign. On the bases of collected data from the parameters of model, we proved that the rate of vaccination for SARS-CoV-2 control spread of disease efficiently.

In near past, the researchers [52] of the fields observe that this virus change their behavior each time. As we know that this virus change their behavior time to time. We are trying to develop a new model for observed new pattern as like beta and omicron in future.

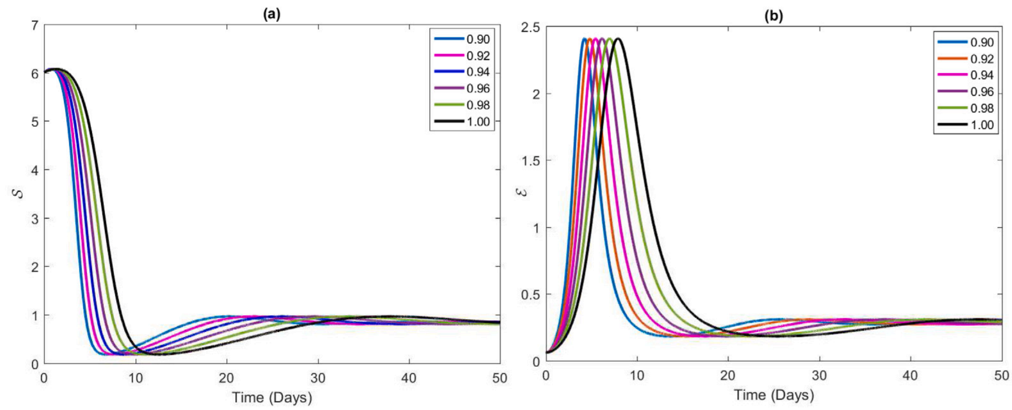

Fig. 6. Dynamical behaviors of susceptible and exposed classes of the considered model corresponding to fractional order model.

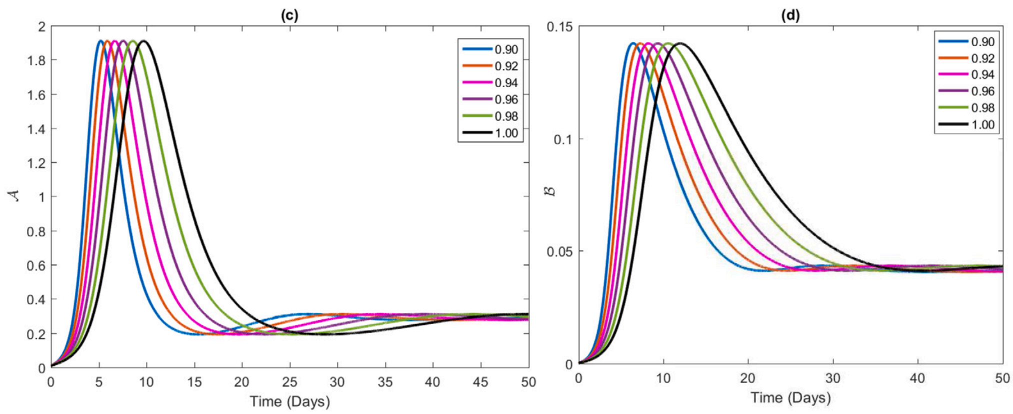

Fig. 7. Dynamical behaviors of symptomatic and symptomatic with no visible symptoms classes of the considered model corresponding to fractional order model.

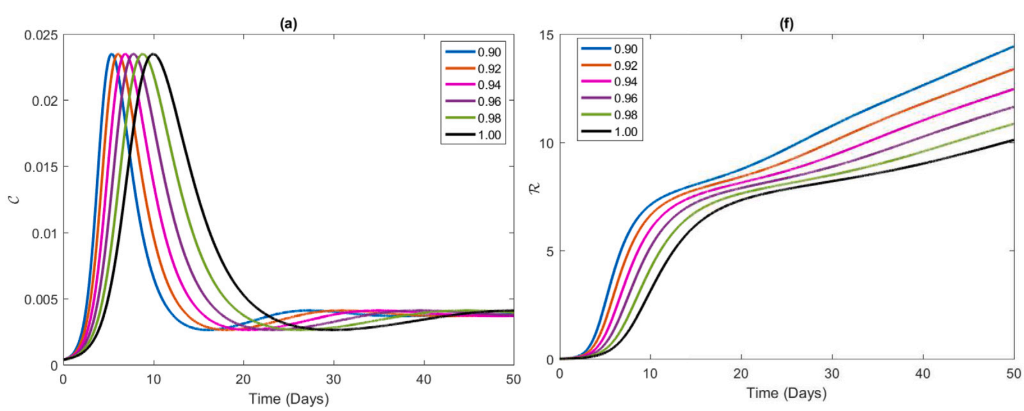

Fig. 8. Dynamical behaviors of omicron infected and recovered classes of the considered model corresponding to fractional order model.

CRediT authorship contribution statement

Hussam Alrabaiah: Theoretical part. **Rahim Ud Din:** Drafted the paper. **Khursheed J. Ansari:** Done the numerical part. **Ateeq ur Rehman Irshad:** Edited and revised the last draft. **Burhanettin Ozdemir:** Edited and revised the last draft.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

No data was used for the research described in the article.

Acknowledgments

The authors extend their appreciation to the Deanship of Scientific Research at King Khalid University, Saudi Arabia for funding this work through large group Research Project under Grant number RGP2/371/44. In addition, authors Ateeq ur Rehman Irshad and Burhanettin Ozdemir are thankful to Prince Sultan University for paying the APC and support through TAS research lab.

References

[1] Omicron Variant: What You Need to Know. https://www.cdc.gov/coronavirus/2019-ncov/variants/omicron-variant.html.
[2] World Health Organization (WHO). Naming the coronavirus disease (SARS-CoV-2) and the virus that causes it. 2020, Archived from the original on 28 2020. Retrieved 28 February, 2020.
[3] Hui DSI, Madani Azhar E, Ntoumi TA, Kock F, Dar R, Ippolito O, et al. The continuing 2019-nCoV epidemic threat of novel coronaviruses to global health-the latest 2019 novel coronavirus outbreak in Wuhan, China. Bull Math Biol 2020;91(6):264–6.
[4] Zhao S, Lin Q, Ran J, Musa SS, Yang G, Wang W, et al. Preliminary estimation of the basic reproduction number of novel coronavirus (2019-nCoV) in China. Int J Infect Dis 2020;92:214–7.
[5] Zhao S, Musa SS, Lin Q, Ran J, Yang G, Wang W, et al. Estimating the unreported number of novel coronavirus (2019-nCoV) cases in China in the first half of 2020, a data-driven Modelling analysis of the early outbreak. J Clin Med 2020;9(2):388.
[6] Shah K, Din RU, Deebani W, Kumam P, Shah Z. On nonlinear classical and fractional order dynamical system addressing COVID-19. Results Phys 2021;24:104069.
[7] Din A, et al. Mathematical analysis of spread and control of the novel corona virus (COVID-19) in China. Chaos Solitons Fractals 2020;141:110286.
[8] Goel NS, Maitra SC, Montroll EW. On the Volterra and other nonlinear models of interacting populations. Rev Modern Phys 1971;43(2):231.
[9] Khan MA, Atangana A, Alzahrani E. The dynamics of COVID-19 with quarantined and isolation. Adv Difference Equ 2020;2020(1):1–22.
[10] Khan H, Gómez-Aguilar JF, Alkhazzan A, Khan A. A fractional order HIV-TB coinfection model with nonsingular Mittag-Leffler Law. Math Methods Appl Sci 2020;43(6):3786–806.
[11] Bogoch II, Watts A, Thomas-Bachli A, Huber C, Kraemer MU, Khan K. Pneumonia of unknown aetiology in Wuhan, China: potential for international spread via commercial air travel. J Travel Med 2020;27(2):taaa008.
[12] Wu JT, Leung K, Leung GM. Nowcasting and forecasting the potential domestic and international. Lancet 2020;395(10225):689–97.
[13] Lopez Leonardo, Rodo Xavier. A modified SEIR model to predict the COVID-19 outbreak in Spain and Italy: simulating control scenarios and multi-scale epidemics. Results Phys 2021;21:103746.
[14] Karthikeyan Kulandhivel, Karthikeyan Panjaiyan, Baskonus Haci Mehmet, Venkatachalam Kuppusamy, Chu Yu-Ming. Almost sectorial operators on Ψ-hilfer derivative fractional impulsive integro-differential equations. Math Methods Appl Sci 2021;45(13):8045–59.
[15] Anwarud Din, et al. On Analysis of fractional order mathematical model of Hepatitis B using Atangana-Baleanu Caputo (ABC) derivative. Fractals 2022;30(1):2240017.
[16] Jin Fang, Qian Zi-Shan, Chu Yu-Ming, ur Rahman Mati. On nonlinear evolution model for drinking behavior under Caputo–Fabrizio derivative. J Appl Anal Comput 2022;12(2):790–806.
[17] Gu Y, Khan MA, Hamed YS, Felemban BF. A comprehensive mathematical model for SARS-CoV-2 in Caputo derivative. Fract Fract 2021;5(4):271.
[18] Khan A, Abdeljawad T, Gomez-Aguilar JF, Khan H. Dynamical study of fractional order mutualism parasitism food web module. Chaos Solitons Fractals 2020;134:109685.
[19] Khan A, Alshehri HM, Gómez-Aguilar JF, Khan ZA, Fernández-Anaya G. A Predator–Prey model involving variable-order fractional differential equations with Mittag-Leffler kernel. Adv Difference Equ 2021;2021(1):1–18.
[20] Khan A, Alshehri HM, Abdeljawad T, Al-Mdallal QM, Khan H. Stability analysis of fractional nabla difference COVID-19 model. Results Phys 2021;22:103888.
[21] Ahmad S, Ullah A, Al-Mdallal QM, Khan H, Shah K, Khan A. Fractional order mathematical modeling of COVID-19 transmission. Chaos Solitons Fractals 2020;139:110256.
[22] Khan H, Tunç C, Khan A. Stability results and existence theorems for nonlinear delay-fractional differential equations with φ_p^*-operator. J Appl Anal Comput 2020;10(2):584–97.
[23] Aldila Dipo. Analyzing the impact of the media campaign and rapid testing for COVID-19 as an optimal control problem in East Java, Indonesia. Chaos Solitons Fractals 2020;141:110364.
[24] Sun Deshun, Duan Li, Xiong Jianyi, Wang Daping. Modeling and forecasting the spread tendency of the COVID-19 in China. Adv Difference Equ 2020;2020(1):1–16.
[25] Gao Wei, Veeresha P, Baskonus Haci Mehmet, Prakasha DG, Kumar Pushpendra. A new study of unreported cases of 2019-nCOV epidemic outbreaks. Chaos Solitons Fractals 2020;138:109929.
[26] Erturk Vedat Suat, Kumar Pushpendra. Solution of a COVID-19 model via new generalized Caputo-type fractional derivatives. Chaos Solitons Fractals 2020;139:110280.
[27] Nabi KN, Abboubakar H, Kumar P. Forecasting of COVID-19 pandemic: From integer derivatives to fractional derivatives. Chaos Solitons Fractals 2020;141:110283.
[28] Kumar P, Suat Erturk V. The analysis of a time delay fractional covid-19 model via caputo type fractional derivative. Math Methods Appl Sci 2020;46(7):7618–31.
[29] Kumar P, Erturk VS, Abboubakar H, Nisar KS. Prediction studies of the epidemic peak of coronavirus disease in Brazil via new generalised Caputo type fractional derivatives. Alex Eng J 2021;60(3):3189–204.
[30] Nabi KN, Kumar P, Erturk VS. Projections and fractional dynamics of COVID-19 with optimal control strategies. Chaos Solitons Fractals 2021;145:110689.
[31] Khan Muhammad Altaf, Atangana A. Dynamics of Ebola disease in the framework of different fractional derivatives. Entropy 2019;21(3):303.
[32] Zeb Anwar, Kumar Pushpendra, Erturk Vedat Suat, Sitthiwirattham Thanin. A new study on two different vaccinated fractional-order COVID-19 models via numerical algorithms. J King Saud Univ Sci 2022;34(4):101914.
[33] Din A, Li Y, Yusuf A, Ali AI. Caputo type fractional operator applied to Hepatitis B system. Fractals 2022;30(01):2240023.
[34] Nazir Ghazala, Zeb Anwar, Shah Kamal, Saeed Tareq, Khan Rahmat Ali, Khan Sheikh Irfan Ullah. Study of COVID-19 mathematical model of fractional order via modified Euler method. Alex Eng J 2021;60(6):5287–96.
[35] Zamir Muhammad, Shah Kamal, Nadeem Fawad, Bajuri Mohd Yazid, Ahmadian Ali, Salahshour Soheil, et al. Threshold conditions for global stability of disease free state of COVID-19. Results Phys 2021;21:103784.
[36] Khan MA, Atangana A, Alzahrani E. The dynamics of COVID-19 with quarantined and isolation. Adv Difference Equ 2020;2020(1):425.
[37] Ali Zeeshan, Rabiei Faranak, Shah Kamal, Khodadadi Touraj. Qualitative analysis of fractal-fractional order COVID-19 mathematical model with case study of Wuhan. Alex Eng J 2021;60(1):477–89.
[38] ud Din Rahim, Seadawy Aly R, Shah Kamal, Baleanu Aman Ullah Dumitru. Study of global dynamics of COVID-19 via a new mathematical model. Results Phys 2020;19:103468.
[39] Subhas Khajanchi, Sarkar Kankan, Mondal Jayanta, Nisar Kottakkaran Sooppy, Abdelwahab Sayed F. Mathematical modeling of the COVID-19 pandemic with intervention strategies. Results Phys 2021;25:104285.
[40] Van den Diressche P, Watmough J. Reproduction number and sub threshold equilibria for compartmental models of disease transmission. Math Biosci 2002;180(1):29–48.
[41] Wang B, Li L, Wang Y. An efficient nonstandard finite difference scheme for chaotic fractional-order Chen system. IEEE Access 2020;8:98410–21.
[42] Arenas AJ, González-Parra G, Chen-Charpentier BM. Construction of nonstandard finite difference schemes for the SI and SIR epidemic models of fractional order. Math Comput Simulation 2016;121:48–63.
[43] Morakaladi MIC, Atangana A. Mathematical model for conversion of groundwater flow from confined to unconfined aquifers with power law processes. Open Geosci 2023;15(1):20220446.
[44] Riaz MB, Rehman AU, Wojciechowski A, Atangana A. Heat and mass flux analysis of magneto-free-convection flow of Oldroyd-B fluid through porous layered inclined plate. Sci Rep 2023;13(1):1–15.
[45] Khan A, Abdeljawad T, Khan H. A numerical scheme for the generalized ABC fractional derivative based on Lagrange interpolation polynomial. Fractals 2022;30(05):2240180.
[46] Hadian Rasanan AH, Rad JA, Tameh MS, Atangana A. Fractional Jacobi Kernel functions: Theory and application, in learning with fractional orthogonal kernel classifiers in support vector machines: theory, algorithms and applications (119–144). Singapore: Springer Nature Singapore; 2023.
[47] Abro KA, Atangana A, Gómez-Aguilar JF. A comparative analysis of plasma dilution based on fractional integro-differential equation: an application to biological science. Int J Model Simul 2023;43(1):1–10.
[48] Watson Oliver J, Barnsley Gregory, Toor Jaspreet, Hogan Alexandra B, Winskill Peter, Ghani Azra C. Global impact of the first year of COVID-19 vaccination: a mathematical modelling study. Lancet Infect Dis 2022;22(9):1293–302.
[49] Sam Moore, Hill Edward M, Tildesley Michael J, Dyson Louise, Keeling Matt J. Vaccination and non-pharmaceutical interventions for COVID-19: a mathematical modelling study. Lancet Infect Dis 2021;21(6):793–802.
[50] Yavuz M, Cosar FÖ, Günay F, Özdemir FN. A new mathematical modeling of the COVID-19 pandemic including the vaccination campaign. Open J Model Simul 2021;9(3):299–321.
[51] Fu ZJ, Tang ZC, Zhao HT, Li PW, Rabczuk T. Numerical solutions of the coupled unsteady nonlinear convection–diffusion equations based on generalized finite difference method. Eur Phys J Plus 2019;134(6):1–20.
[52] Ouncharoen R, Shah K, Ud Din RAHIM, Abdeljawad T, Ahmadian A, Salahshour S, et al. Study of integer and fractional order COVID-19 mathematical model. Fractals 2023;31(4):2340046.