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Does flipping the classroom with videos and notetaking promote high school students' performance in mathematics?

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Abstract

Using a quasi-experiment with the pre-/post-test control group design, the present study investigated whether flipping an advanced pre-calculus class would lead to a statistically significant gain in the learning of conic sections for high school students. The subjects were 50 11th graders (n=50), who were equally divided into two groups. The treatment group (n=25) learned conic sections via the flipped classroom model, while the control group (n=25) was taught the same topic traditionally via the didactic approach. The intervention comprised flipping mathematics class with four video-assisted lessons and notetaking. All subjects were pre-tested on their prior knowledge on conic sections before starting the experiment and post-tested after intervention. The results of the post-test indicated statistically significant difference between the mean score of the treatment group and that of the control group, reflecting the effectiveness of the flipped instruction. Participants enjoyed the intervention.

Keywords: flipped classroom, flipped instruction, mathematics performance, technology-supported flipped learning, student-centered active learning, quasi-experimental study

INTRODUCTION

Flipped Classroom Concept and Features

Flipped learning network (https://flippedlearning.org/) defines flipped learning or the flipped classroom as a pedagogical approach that transforms classroom learning from a teacher-centered direct instruction into a student-centered active learning process. The approach has also been described as an educational model in which the standard lecture and homework elements of a course are "reversed" or "flipped" (Moraros et al., 2015, p. 2). In this alternative set-up, the transmission of knowledge is intentionally designed to occur before the class meeting, either with or without the use of digital resources such as videos, pdf notes, PowerPoint slides, websites, podcasts, and the like, while actual class time is spent on clarifying concepts, asking and answering questions, solving problems, and discussing answers, errors, or solutions. In other words, what was "traditionally done in class is now done at home, and what was traditionally done as homework is now completed in class" (Bergmann & Sams, 2012, p. 13).

The primary reasons for implementing such a learning design are to increase deep learning among students, empower them to acquire knowledge on their own, and shift the learning responsibility to students, making them responsible for their own learning. From the cognitive perspective, a flipped instruction increases students' ability to understand new content through an active processing of the content. Recent studies also show that flipped learning could significantly improve content mastery (Hew & Lo, 2018) and conceptual understanding (Kirvan et al., 2015), especially among low prior-knowledge students, and higher-order thinking skills in terms of application, analysis, and evaluation (Almasseri & AlHojailan, 2019). Teachers can pave the way for students to develop these skills–largely through their own effort–by providing the right resources, stimuli, and scaffolds. In a non-technology assisted flipped classroom, the learning materials or resources may come in the form of handouts, a short story, or a book chapter to read before class. **Figure 1** shows the structure of the flipped classroom and sample activities.

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Contribution to the literature

- This study contributes to the understanding of how videos and note-taking sheets are adapted when using a flipped classroom approach. In the UAE, flipped instruction is still in its infancy.
- This is one of very few empirical studies in the UAE that investigates the effect of a flipped classroom approach on teaching mathematics for high school curriculum in UAE.
- As an Arab country, the findings of this study have implications not only for the UAE but other Arab countries that are keen to integrate technology in their classrooms.

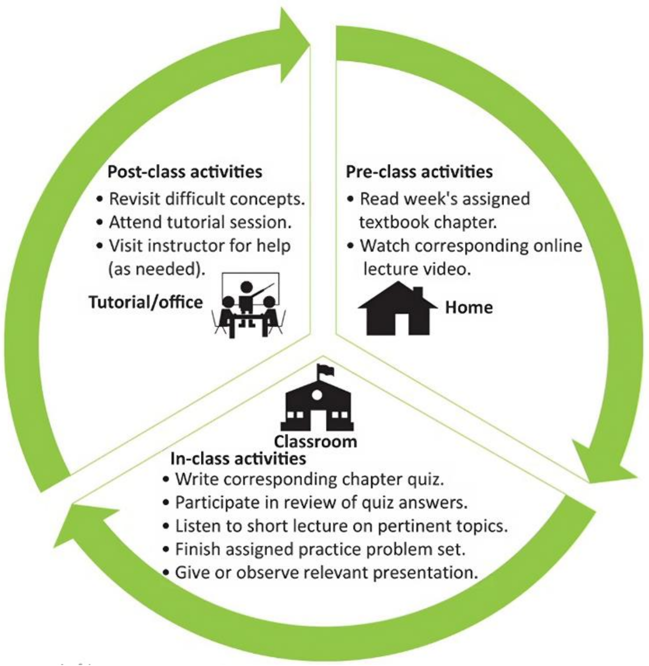

Figure 1. Flipped classroom structure & sample activities (Moraros et al., 2015)

To ensure the success of flipped instruction, teachers must be very careful in choosing the pre-class reading materials and in designing the corresponding learning tasks. They must establish a close alignment between the selected materials and tasks and the intended learning outcomes. For video materials, teachers must ascertain that the content is neither too difficult nor too technical for students to grasp on their own. A good strategy, according to Bergman (as cited in Raths, 2013) himself–the founder of the approach–would be "to keep the lower-order things on Bloom's taxonomy to the videos and the higher-order things in class" (Raths, 2013). Hence, underlying the flipped classroom approach is the principle that teachers should not spend valuable class time on explicitly teaching students what they can acquire on their own. The choice and the matching of pre-class activities are, therefore, critical to achieving the ultimate aim of flipped instruction.

Flipping a Mathematics' Class With Videos

A precondition to successful flipped learning, generally, is to use well-prepared lecture videos, notes, slides, and teaching materials for pre-class study activities (Chen, 2016; Lo & Hew, 2017).

Currently, the most popular strategy among teachers, especially in post-pandemic flipped online teaching, is to have students watch a video related to the lesson before the actual class meeting. Students benefit a great deal from well-selected videos but for mathematics lessons, choosing the right videos is a real challenge for teachers (Holmes at al., 2015). The difficulty lies in finding topic-specific content that closely aligns with mathematics syllabus or curriculum in use. In particular, it is difficult to find ready-made lecture videos with content that fully meets the expectations of teachers and the needs of students. Although there is a plethora of videos on popular sites such as YouTube, Khan Academy, Mashup Math, and Math Playground, and on less popular ones like Woo Tube, their coverage frequently falls short of the content taught in school (Chen, 2016). Teachers of mathematics can solve this problem by creating their own instructional videos for their flipped classes, but the process would be too time-consuming. Furthermore, most teachers do not have the technical skills to create customized videos. An alternative at hand is to use instructional videos from education-based companies, like Cengage Learning, where mathematics' contents are more aligned with mathematics curriculum as these companies also produce textbooks to be used by schools.

Even if teachers succeed in finding the right videos, a critical question lingers–do students know what to do when watching the videos assigned for their flipped learning? The mistake teachers make is they often assume that students know precisely how to watch a video in the way that promotes learning and prepares them for in-class discussion. Bergman (as cited in Raths, 2013) reminded teachers that they must teach their classes the proper way of watching videos for flipped learning, emphasizing the importance of spending some time going over the basics with them. As instructional videos cannot be watched in the same manner as movies, teachers are advised to give class tutorials on how to extract important content from a video by pausing at the right places and rewinding for information. This is an important skill that all students must have in order to maximize their gain from a video-assisted flipped classroom.

Learning to watch videos in this manner is said to reduce students' cognitive load because they can control the amount of information they can process at a time, thereby allowing them to move at their own pace.

Teachers can further help students in this information process by selecting videos with effective instructional design features, and a high mathematical quality. To maximize learning in a flipped instruction, the study videos must be short (i.e., not longer than 10 minutes), well-segmented, coherent, use language that is precise and appropriate for the target group of learners, not contain unmitigated mathematical errors, and interactive (i.e., by means of asking questions to students while explaining the key ideas) (Otten et al., 2020).

Pairing Videos with Notetaking

Simply watching pre-class videos without fully knowing the purpose of watching and what to take away from them will not prepare students well for a fruitful discussion in a flipped learning environment. Students need to know how to take good notes from the videos. Notetaking is a cognitive strategy that boosts content learning, and this has been proven by decades of research across various disciplines (e.g., Chang & Ku, 2014; Kiewra, 2002; Rahmani & Sadeghi, 2011), including mathematics (Cardetti et al., 2010; Dundar, 2015; Swenson, 2018). Notetaking works because it requires real effort and forces students to focus on the learning material. The process itself is cognitively rigorous as students must decode and encode information, converting it into knowledge forms that they can comprehend with their existing schemata. The mental activities involved in creating good notes render notetaking a beneficial active learning strategy for any class, flipped or traditional.

Dundar (2015) wrote that "note-taking in mathematics classes is important in terms of the form and the process" (p. 1). By this, she meant that not only is the process of taking notes important, but the form of students' notes too, is equally influential in determining the degree of their success in learning mathematics. The quality of notes that students take–i.e., whether they can capture relevant points in their notes, enhancing them with contextual properties (like highlighting and writing side notes), and leave out irrelevant details–is strongly correlated with their mathematics' performance (Dundar, 2015).

To enable students to construct better notes and to develop their notetaking skills in a gradual process, teachers should build scaffolds into their instruction. Teachers can be creative in creating the scaffolds, but normally they come in the form of skeletal or guided notes containing an outline of the content to be learned, leaving enough space for students to complete key information. Providing partially completed or scaffolded notes has been shown to help students get the most out of everyday lessons, thereby increasing their achievement and performance in school subjects substantially across all grade levels (Haydon et al., 2011).

A high school mathematics teacher in the US, with a passion for flipped learning, Crystal Kirch, developed a notetaking strategy for students to use with video watching, which she called the WSQ ("wisk") framework. The acronym stands for watch, summarize and question. The strategy teaches students to use the pause and rewind buttons strategically while watching videos to make sure they fully grasp the content and then jot down key ideas. In the second step, they must complete a summary of the lesson using a Google Form that is placed right below the video. Finally, each student must come up with a question to bring to class. The question can be general or specific but must be related to any part of the lesson that they may still be confused about. Crystal Kirch's strategy is helpful in increasing students' cognitive engagement with and active processing of the content and should be considered by teachers if they wish to enhance students' performance in mathematics.

Theoretical Framework

How the flipped classroom approach works to promote student learning can be explained by the principles of constructivism. Constructivists assert that students learn best when they construct their own understanding and knowledge of the content to be learned through an active engagement with or an active processing of the learning material. Such an engagement or processing triggers deep thinking in students' minds (i.e., by means of asking questions and looking for examples they can relate to) and encourages them to connect the new knowledge with what they already knew (Bereiter, 1994).

Constructivism rejects the idea of learning through passively listening to teacher talk, copying notes mindlessly, memorizing facts without understanding, or simply accepting the authoritative views of the teacher (or anyone else for that matter). Constructivist methods invite teachers "to pay close attention to the mental activities of the learner" (Bereiter, 1994, p. 21), denouncing the claim that students' minds are tabula rasa that must be filled with knowledge and information constructed by others. Constructivist pedagogy emphasizes active interaction among elements in the learning environment, i.e., the student, the teacher, peers, the learning material, and resources). In short, the theory believes in learning as an active-interactive process of meaning making, which must be done by the student himself/herself. Teachers can facilitate this process by providing the right combination of resources, stimuli, scaffolds and instructional guidance.

Based on the preceding principles, the flipped classroom can be viewed as a constructivist learning environment, where all the elements in it contribute to the active construction of knowledge by students and to the progressive development of their content understanding. Teachers galvanize and facilitate

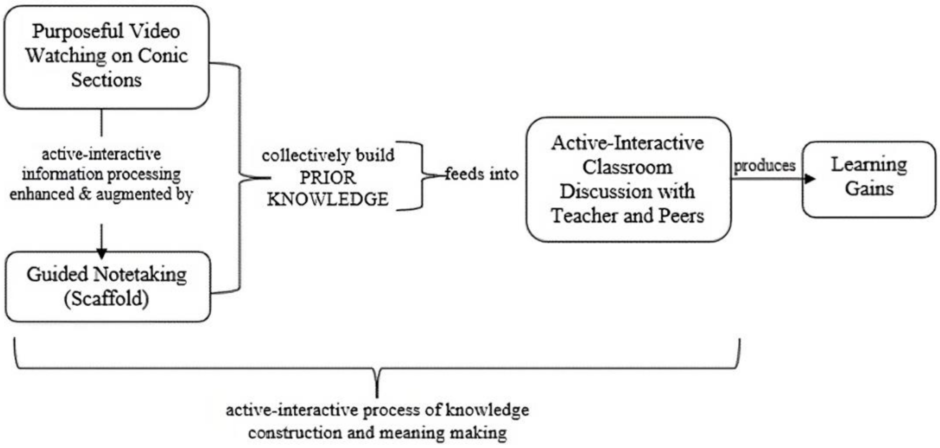

Figure 2. A constructivist perspective of how study's flipped learning environment works to produce learning gains (Source: Authors' own elaboration)

students' active processing and construction of knowledge through resources like videos and scaffolds like note sheets for students to use in a guided notetaking activity. With these pre-class study activities, students come to class meetings equipped with a substantial amount of prior knowledge to enable them to participate meaningfully in the class discussion on the content. Students ask questions to clarify their lack of understanding and interact with the teacher and peers to make meaning of the new content. The constructivist process of learning mathematics, embodied in the present study's flipped learning environment is visualized in **Figure 2**.

LITERATURE REVIEW: EFFECTS OF THE FLIPPED CLASSROOM

Results of Selected Individual Studies

Mathematics teachers around the world are increasingly adopting and innovating the flipped classroom approach–enhancing it with a combination of myriad strategies–in the hope of improving mathematics' understanding and performance of their students. Since its introduction in 2007 by Jonathan Bergman and Aaron Sams, the pedagogical approach has been tested, proven, and disproven in scores of empirical studies. While many researchers found a positive effect of flipping the classroom, a few others reached conclusions that are quite the opposite.

For example, in a quasi-experiment involving 88 grade 6 students in a Chinese secondary school, the proposed flipped classroom intervention significantly improved the subjects' mathematics' learning performance. The researchers discovered that the flipped approach benefitted students at the moderate mathematics' proficiency level more than it did those at the high or low level (Wei et al., 2020). Students in the low-achievement category did not perform as well as expected, reacting that they could not watch the assigned videos due to poor self-control and the videos' lack of appeal. This disclosure informs us that teachers need to consider the instructional design appeal of videos before choosing them as pre-class study materials. Selecting short, attractive, and content relevant videos may help students to increase their attention span and overcome their problem of poor self-control in an autonomous learning setting.

Using a larger sample of 275 high school students, a Nigerian study concluded that flipping the classroom improved student retention and "encourage[d] good performance in mathematics" (Makinde, 2020, p. 23), attributing the success of the approach to students' active engagement in classroom activities and discussion. The success factor explained by Makinde (2020) reflects the position of cognitive and constructivist theorists about how meaningful and deep learning of content should occur. Earlier studies recorded similar positive results (e.g., Bhagat et al., 2016; Casem, 2016). The flipped learning environment in Bhagat et al. (2016) resulted in the experimental group's superior performance in trigonometry and benefited students in the low-ability category, especially. Bidwell (2014) also reported that the collective performance of low achievers in the flipped classroom was 10% higher than the performance of low achievers in the traditional classroom. These earlier observations ran contrary to the recent findings of Wei et al. (2020) whose low-ability group did not benefit as expected from the flipped instruction. To understand these differences in student performance requires looking at each study's experimental design, characteristics of the subjects, and most importantly, the design of the flipped classroom as an instructional intervention.

In Casem (2016), both the treatment and control groups demonstrated the same level of performance on mathematics post-test given after the flipped learning intervention–the t-test results did not yield a statistically significant difference in the groups' mean scores. The researcher concluded that at face value, both groups were therefore comparable in achievement. However, the flipped classroom approach had managed to produce a higher gain score in the treatment group (M=13.08) than in the control group (M=10.67), at a very large effect size of Cohen's d=0.9. Thus, although they appeared to be comparable in terms of test scores, the flipped learning design had actually closed the performance gap between the control and treatment groups. The researcher concluded that

> "[al]though the two groups performed basically at the same level in the pre- and post-test, the significant difference [in] their gain scores suggests that those participants in the experimental group improved better than those in the control group. This implies that the use of flipped instruction had a positive effect on mathematics performance of the participants in the experimental group" (p. 41).

The positive outcomes of flipping the classroom lend an empirical basis that can be used to support mathematics teachers' decision to adopt the approach to improve the overall teaching and learning. According to Cronhjort et al. (2018), flipped learning not only boosted students' performance in mathematics', but it also decreased the failure rate among those that received the intervention. They concluded that the approach "was beneficial for low as well as high-performing students" (p. 119). In a two-cycle action research involving 130 grade 8 students, Lo (2017) also concluded that the flipped classroom approach has a definite positive impact on students' achievement and emphasized how important it is for the study materials and resources to "be well prepared by teachers and given ahead of time" (p. 135) to students. Flipped-learning can also help students learn more effectively by controlling the pace of their learning. This is because it allows students to present information in a way that works for them (Rizos et al., 2023).

Although encouraging, some of the results should be read with caution as some of the studies with positive results are rather constrained in the generalizability of their findings due to poor research design. For instance, Ramakrishnan and Priya (2016) documented a significant increase of 11 points in their flipped subjects' mathematics test scores, but due to the lack of a control group, their finding is limited in terms of applicability across subject matter, settings, and student groups. Hence, teachers and researchers should exercise caution in using the results of empirical studies to inform their classroom practices and decision-making, particularly in regard to flipping the classroom.

It is interesting to see quite a number of studies with results that have challenged the positive outcomes of flipped learning, finding no significant difference in student performance between the flipped classroom and the traditional classroom (Chen, 2016; Clark, 2015; DeSantis et al., 2015; Kirvan et al., 2015; Ramaglia, 2015). Ramaglia (2015), for instance, reported no significant effects of flipped learning between groups despite the active learning that took place in the treatment group. The researcher explained that due to the classes' different physical settings and seating arrangements, more active learning actually occurred in the control group, and this might have skewed the results in favor of traditional instruction.

Worth discussing are the results of Cabi (2018), which did not align with the previously mentioned positive findings. In fact, her research found a decrease in the flipped subjects' test performance–similar to that found by Kirvan et al. (2015)–in addition to discovering no significant achievement difference between the flipped classroom and the traditional method groups. In a post-experiment interview, some of the treatment subjects expressed that they preferred "to learn [the lesson] from the instructor [himself]" (p. 214), instead of from the videos. Others said that the topic was difficult to understand without teacher help and the fact that too many technical terms pervaded the assigned videos confounded their understanding even further. Generally, some resistance was expressed by the students as they felt "forced to learn" the content on their own outside the classroom in the flipped learning approach. Another student wrote that having a "serious" learning environment at home, which he/she did not have, was important in supporting flipped learning. Cabi's (2018) findings suggest that the outcomes of the flipped classroom are dependent on a multitude of factors, including the quality of students' home environment, and not just on using the right videos and having a good discussion in class. Part of her findings are comparable to those of Cevikbas and Kaiser (2020), where students' resistance to flipping the learning structure had created a substantial amount of difficulty for the teacher, hence discouraging her from using the flipped classroom approach in teaching mathematics.

Results of Selected Meta-Analyses

The mixed outcomes of flipping the classroom found in individual studies have been documented and confirmed in several meta-analyses, which also detected varying effect sizes of the intervention when positive effects were recorded. The effect sizes depended on the variability in the studies' research design, subjects, flipped learning strategies and context. Overall, flipped classroom interventions in their heterogenous forms produced positive gains across the cognitive learning domain and improved student performance with varying degrees of practical importance (i.e., with effect sizes ranging from g=0.20 to g=0.53). Several moderators appeared to influence the relative efficacy of flipped courses, namely discipline of study, data collection

Table 1. Schematic representation of study's pre-/post-test control group design

Random assignment	Pre-test	Treatment (flipped learning)	Post-test	Treatment group
Random assignment	Pre-test	⟶	Post-test	Control group

Table 2. Total population of AIS High School students (campus one) by grade level (n=887)

Grade level	Frequency (n)	Percentage (%)
Grade 6	126	14.2
Grade 7	126	14.2
Grade 8	123	13.9
Grade 9	130	14.7
Grade 10	118	13.3
Grade 11	151	17.0
Grade 12	113	12.7
Total population	887	100.0

methods and school location, with educational context accounting for the most variability in the learning outcomes. These patterns were observed based on data from 317 studies (Bredow et al., 2021).

Meanwhile, using the results of 55 studies, the meta-analysis of Cheng et al. (2019) yielded an even smaller effect size of the flipped classroom intervention (g=0.193; p<.001), lower than that discovered by Lo et al. (2017) who examined 21 studies in mathematics education and found an effect of g=0.30, which was more comparable to that recently documented by Bredow et al. (2021). A reasonable conclusion that can be drawn from the selected meta-analyses is–despite the mixed effects, the flipped classroom intervention is more likely to produce positive outcomes, rather than negative ones, when applied in teaching mathematics. And even when a statistically significant improvement in student performance is found, the effect size of the improvement is more likely to be small or moderate, rather than large.

Research Problem, Objective, and Hypothesis

Although flipped instruction is touted as an innovation that can reform and improve modern day mathematics' teaching and learning (Cevikbas & Kaiser, 2020; Montgomery, 2015), previous studies have shown mixed results on the effects of the approach on academic achievements and performance. This was due to the influence of various moderating factors, most especially the strategies that researchers incorporated into their flipped learning intervention and the quality of resources used. Some studies had used only video watching, while others had integrated cognitive strategies like notetaking and question writing as part of the intervention. Others had utilized videos that students found difficult to learn from.

Hence, our main purpose for conducting the present study was to shed light on this confounding situation on the goodness of the flipped classroom approach–by examining its effect on students' performance in mathematics. Using a pre-/post-test control group quasi-experiment with grade 11 students, we tested the following research hypothesis:

H1. The treatment group will produce a significantly superior mathematics' performance than the control group because of the flipped classroom intervention.

METHODS

Research Design

A pre-/post-test, quasi-experimental control-group design was used in our research to study the effects of the flipped classroom approach on grade 11 students' ability to solve questions on conic sections, used as a measure of their mathematics' performance. The research design is schematically presented in **Table 1**.

Population and Target Group

Around 2,700 students of different nationalities, distributed over two campuses, were enrolled in the Abu Dhabi International Private School (AIS) in the 2018/2019 academic year. Our quasi-experiment was conducted at one of the campuses with a student body of 1,946, which included pre-school and primary level students. High school starts from grade 6 to grade 12 with a population of 887 students. AIS follows both the American and International Baccalaureate (IB) curricula with English as the medium of instruction. **Table 2** shows the distribution of AIS students by grade level at the campus, where we performed the study.

The study's target population included all grade 11 students in campus one of AIS (n=151) to whom the quasi-experimental results would be applicable.

Subjects

The research subjects were 50 grade 11 students, aged approximately 17, and represented roughly one-third of the target population. They were evenly divided into the treatment group (n=25) and control group (n=25). Both groups were equal in mathematical skills and prior knowledge, with class 11A having an average of 80% and class 11B, 81%. Being in the same grade level and of the same abilities, both groups were studying the same level of mathematics (pre-calculus), focusing on a topic called conic sections. Class 11A was randomly chosen as the control group, while class 11B as the treatment group.

Table 3 shows the gender and class breakdown of the subjects.

Table 3. Gender & class breakdown of the research subjects (n=50)

Group	Class	Boys	Girls	Total
Control	11A	15	10	25
Treatment	11B	11	14	25
Total		26	24	50

Table 4. Content on conic sections & expected learning outcomes

Lesson	Subtopic	Content	Learning outcomes (Students can)
1	Parabolas	Geometric definition of a parabola, equations, & graphs of parabolas & applications	1. find the equation of a parabola 2. find focus & directrix of a parabola from its equation 3. find focal diameter of a parabola 4. graph a family of parabolas
2	Ellipses	Geometric definition of ellipses, equations, & graphs of ellipses & applications	1. find equation of a parabola 2. find focus & directrix of a parabola from its equation 3. find focal diameter of a parabola 4. graph a family of parabolas
3	Hyperbolas	Geometric definition of a hyperbola, equations, & graphs of hyperbolas	1. graph hyperbolas with horizontal transverse axis 2. graph hyperbolas with vertical transverse axis 3. find equation of a hyperbola from its vertices & foci 4. find equation of a hyperbola from its vertices & asymptotes

Learning Objectives

Mathematics' topic taught to both groups in this quasi-experiment was conic sections. The study of conic-section curves is very important as it has many practical applications, for example in creating satellite dishes and light reflectors, in the design of camshaft inside an engine, and many others.

Table 4 outlines the content taught to both groups over four lessons and the expected learning outcomes.

Intervention: Flipped Learning for Treatment Group

Four 15-minute videos from Cengage Learning website, that explain parabolas (video 1), ellipses (video 2), hyperbolas (video 3), and shifted conics (video 4) were uploaded onto the group's learning platform "Edmodo". All four videos on the subtopics were posted well ahead of time to give students enough opportunity to study them before class. Students in the treatment group were asked to watch the videos before each class, jot down important notes from the videos, and prepare themselves for the classroom discussions. The flipped learning occurred in the treatment group in the following sequence:

1. *Step 1*: The students watched the assigned video on conic sections at home. The videos were accessible on Edmodo. They could watch the video as many times as they wanted to.
2. *Step 2*: Using the note-taking sheets uploaded on Edmodo, students constructed their own lesson notes about conic sections to enhance their understanding of the video instruction. The notes were part of the preparation for their class discussion as the topic would not be directly taught by the teacher. A sample notetaking sheet is attached as **Appendix A**.
3. *Step 3*: Students attended class and engaged in a whole-group discussions on the video content of conic sections.
4. *Step 4*: Students solved problems related to conic sections, to augment and synthesize their learning of the topic.

INSTRUMENTS

Two instruments, i.e., a test about conic sections and a scoring rubric, were used to measure the subjects' performance in mathematics because of flipped learning.

Mathematics Performance Test on Conic Sections

The researchers prepared a four-item subjective test totaling 20 marks to assess the subjects' ability to understand and solve the problems related to conic sections. The scores were taken as an indicator of their mathematics' learning performance. The test included questions that measured the subjects' performance in finding certain points on the curve of the conic section, like the vertex, foci, and center, and assessed their ability to find the graph axis of a symmetry, asymptote, and directrix.

To ensure the test's content validity, the items were reviewed and validated by a panel of three experts. and were improved according to their comments and recommendations. The test was administered twice in the study, first as a pre-test before the first lesson and then as a post-test after completing the fourth lesson. The test given to both groups is attached as **Appendix B**.

Scoring Rubric

The study used equivalent tests (i.e., pre- and post-test had the exact same questions) to measure the subjects' understanding of the characteristics of the four types of conic sections, and their ability to solve the problems given. Thus, a scoring rubric containing correct answers and scores to be awarded was developed to assess the subjects' performance on the given tests. Students' scores on the tests were graded as follows–as indicated explicitly in the rubric:

1. One mark was awarded for every correct answer in finding any of the points on the conic sections.
2. If there was more than one vertex, then the score was proportional to the number of those vertices.
3. Two marks for finding any of the axis.
4. Three marks for analyzing the equation and determining its type (i.e., parabola, hyperbola, ellipse, or circle).

Procedures

The quasi-experiment started with a pre-test given to students in both groups before starting the first lesson on conic sections. Altogether, there were four lessons under this unit. The subjects mostly did not have any prior knowledge of conic sections because the topic was introduced only in grade 11 in Abu Dhabi schools that were following the American curriculum. The study of the unit took approximately two weeks to complete for both groups.

After pre-test, the treatment group followed the flipped learning structure described in the intervention. Students in the control group were not exposed to any teaching strategy other than the traditional, teacher-centered method that they were exposed to in everyday lessons. They were not as aware of the lesson objectives as students in the treatment group and learned the content by listening to teacher explanations about conic sections and how to problem solve them. In class, they frequently asked questions to clarify concepts and the procedures of problem solving.

After finishing the whole unit, which consisted of four lessons and 20 learning outcomes, the students sat for the post-test, comprising the same questions given earlier in the pre-test.

Validity and Reliability

Internal validity of the outcomes

Both classes studied conic sections with the same teacher. Except for the intervention, they were exposed to the same learning and testing conditions. For example, they had the same physical classroom environment and setting (i.e., the teacher used the same homeroom to teach both groups), studied the same material, took the same mathematics' performance test (as pre- and post-test), were evaluated using the same rubric, and finished the conics unit at the same time. The two groups did both pre- and post-test simultaneously in the same exam hall. They were subjected to a highly controlled experimental conditions to control the spurious effects of extraneous variables on the dependent measures (i.e., students' performance in mathematics), thereby establishing the internal validity of the quasi-experiment and its outcomes.

Validity of the measures

Items in mathematics performance test, used as pre- and post-test, were content validated by three experts to ensure that they accurately assessed students' ability to solve problems related to conic sections. The scoring rubric was also checked for accuracy by expert mathematics teachers at the school. Hence, the measures of the students' performance in mathematics used in the quasi-experiment were judged to have content validity, enabling the results to be utilized and interpreted accordingly to reflect the effect of the flipped learning intervention.

Reliability of the measures

The reliability of the data representing the effect of the flipped learning on students' performance in mathematics was ascertained via an inter-rater scoring procedure. The subjects' answer scripts on the conic sections were graded by two mathematics teachers (i.e., by the teacher who taught both groups in the quasi-experiment and his colleague who was also a mathematics teacher at the same school). Their scoring was guided by the rubric created by the researchers. Scores from the two teachers were correlated using the Pearson product moment correlation procedure, yielding a reliability estimate of $r=0.99$ for both pre- and post-test.

Data Analysis

The subjects' performance in mathematics (i.e., their pre- and post-test scores) were converted to percentage points. Their gain scores were calculated by subtracting the pre-test score from the post-test score, and boxplots were used to compare the groups' performances on three measures (i.e., pre-test, post-test, and learning gain). An independent samples *t*-test analysis was performed on the gain scores to ascertain statistically significant differences in the performance of the control and treatment groups. Cohen's d was then computed to estimate the effect size of the difference and to establish the magnitude and practical importance of the flipped learning intervention.

Prior to running the analysis, the assumptions of normality and equality of variance for the independent samples t-test were first checked. Normality of the data distribution for both samples was examined using skewness and kurtosis values, which should range between -1 and +1, while the equal variance assumption was tested using Levene's test (which should not be statistically significant).

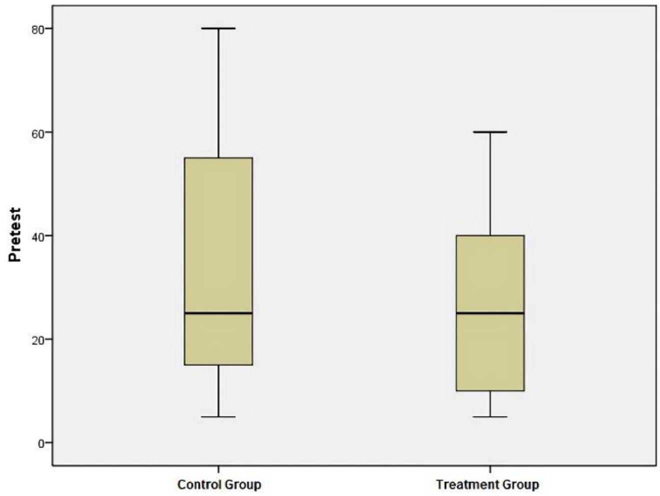

Figure 3. Subjects' performance on pre-test (Source: Authors' own elaboration, using SPSS Statistics software)

RESULTS

Testing of Assumptions

An inspection of the skewness and kurtosis values showed the figures to be reasonably close to zero (i.e., between -.732 and .516 for the control group, and between -.404 and .343 for the treatment group). Therefore, the assumption of normal distribution was met. The Levene's test for equality of variances yielded statistically significant results (F=4.758, p=0.034), meaning that equal variances could not be assumed for the distribution of gain scores across the two groups of subjects. In other words, the assumption of homogeneity of variance was violated, thereby requiring the *t*-test results to be interpreted, where equal variances were not assumed.

Boxplot Analysis of Test Scores

We show the subjects' performance in mathematics on three measures–pre-test, post-test, and the gain score. We can say that the control group started off with a higher mean score (**Figure 3**), indicating their higher prior knowledge on conic sections. The spread of scores was also larger in the control group, indicating greater variability in the prior knowledge of the control group (σ=24.24) than that in the treatment group (σ=16.39).

Figure 4 shows the subjects' post-test achievement, recorded after their treatment conditions. The improvement in performance was very pronounced in the treatment group. Their maximum score increased by 40 points from 60 to 100 points–as opposed to a 20-point increase in the treatment group–and their minimum score improved from five points to 50 points. The spread in scores was also greatly reduced by at least two standard deviations to σ=13.99. This means that flipped learning was able to reduce the performance gap and variability among the treatment group subjects.

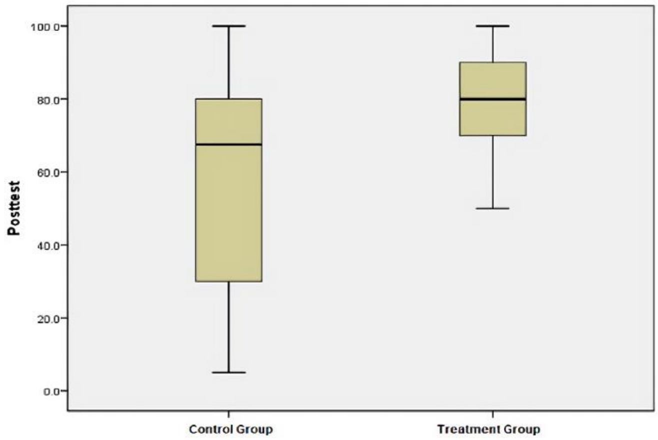

Figure 4. Subjects' performance on post-test (Source: Authors' own elaboration, using SPSS Statistics software)

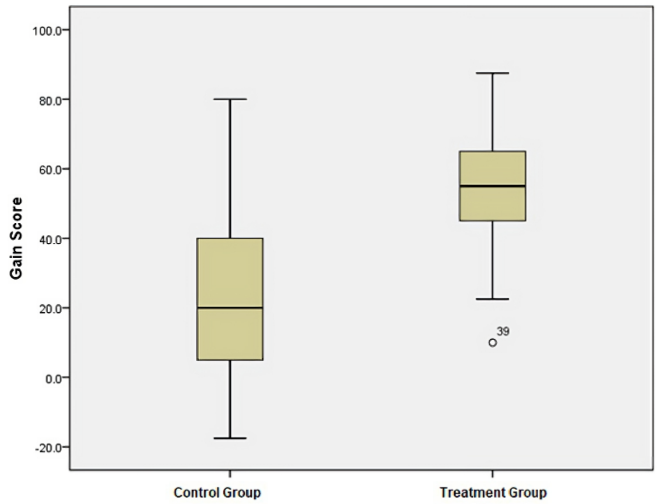

Figure 5. Subjects' gain scores after treatment conditions (Source: Authors' own elaboration, using SPSS Statistics software)

Meanwhile, the performance gap in the control group widened by roughly four standard deviations (σ=28.29).

Figure 5 gives a clearer picture of the effect of the flipped classroom by presenting a visual representation of the subjects' gain scores. The mean, maximum and minimum scores were all higher in the treatment group. Their performance gap was also much smaller than that observed in the control group. In comparison, the performance variation in the control group did not appear to change greatly.

Effect of Flipped Classroom: Independent Samples t-Test Results

As indicated by our analysis of the boxplots, both groups recorded substantial improvements in test results because of their respective treatment conditions. Their respective improvements are summarized in **Table 5**. The control group started off with a higher prior knowledge score (M=32.8) on the topic, but the flipped learning intervention was able to close this gap between the two groups. From the descriptive statistics presented

Table 5. Independent samples t-test results & descriptive statistics of flipped learning intervention (n=50)

Group	n	Mean			σ	df	p
		Pre-test	Post-test	Gain			
Treatment	25	26.0	80.2	54.2	18.7	48	.001
Control	25	32.8	58.1	25.3	28.2		

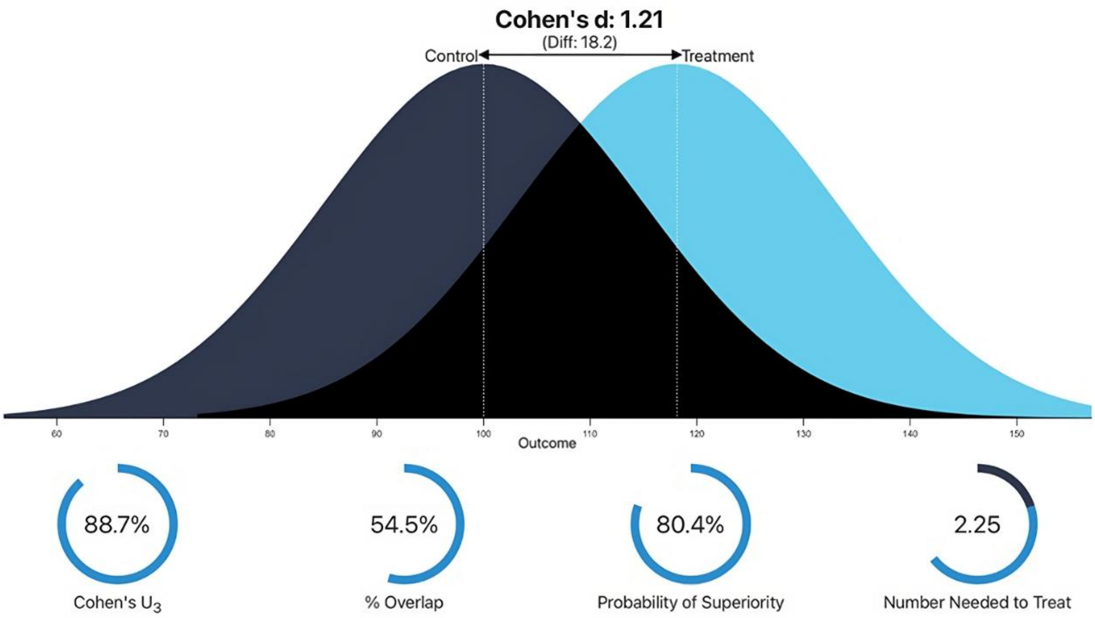

Figure 6. A visual interpretation of meaning of Cohen's d=1.21 (Source: https://rpsychologist.com/cohend/)

in **Table 5**, we can see that the treatment group's performance in mathematics increased by threefold, that is from a 26-point average to an 80.2-point average, while the control group increased by less than twofold.

The control subjects increased their scores by 25.3 points on the average, and the treatment subjects by 54.2 points on the average. The t-test results indicate that the groups' difference in gain scores was statistically significant, t(41.68)=4.272, p=.001. The performance of the treatment group (M=54.2, σ=18.7) was superior to the control group (M=25.3, σ=28.2) by almost 29 points.

Practical Importance and Magnitude of the Flipped Learning Effect

A subsequent effect size analysis of the groups' difference in gain scores produced a Cohen's d of 1.21, suggesting a very large impact of the flipped classroom intervention. A Cohen's d that is bigger than one means that the difference between the two means is larger than one standard deviation. For the present study, the flipped classroom had produced a mean difference of larger than 1.21 standard deviation between the treatment and control groups. **Figure 6** visualizes the meaning of this result.

What the result means is with a Cohen's d of 1.21, 88.7% of the students in the flipped learning group were above the mean of the control group (Cohen's U_3), and there is an 80.4% likelihood that a student picked at random from the flipped learning group would have a higher score than a student, also picked at random, from the control group (i.e., probability of superiority). In addition, to increase the positive outcome in the treatment group compared to the control group, roughly two students on average would have to be treated with flipped learning (Kapur et al., 2022). Based on these statistics, the study's hypothesis (i.e., the treatment group will produce a significantly superior performance in mathematics than the control group because of the flipped classroom intervention) was empirically supported.

DISCUSSION AND CONCLUSION

This study aligns with other studies that indicate a positive impact of flipped learning on students' cognitive outcomes. For example, another recent study (Larson & Sulsky, 2017) found that students who participated in flipped classrooms performed better on traditional tests than their non-flipped peers.

Another example, researchers at the University of Texas at Austin found that students who used a flipped-learning model outperformed their peers by 3% (Cavalier & Kistler, 2011). In addition, a study by the University of Virginia found that students who used a flipped-learning model performed better on tests than those who did not (Dunn, 2013).

The results also confirmed most previous studies (e.g., Cappell & Coppell, 2022; Cronhjort et al., 2018; Lo, 2017; Makinde, 2020; Wei et al., 2020) on the positive impact of flipped learning on students' cognitive outcomes–this was aptly reflected in their substantial gain scores. The effect size of the gain was also very large, i.e., more than one standard deviation, and practically important, suggesting that the flipped instruction was really effective in improving students' learning and performance in mathematics. The effect size produced in the study approximated the practical importance of flipped learning demonstrated by Casem (2016), and challenges the low effects reported in the selected meta-analyses (i.e., Atta & Bonyah, 2023; Bredow et al., 2021; Cheng et al., 2019; Lo et al., 2017; Sopamena et al., 2023). In addition to performing better in mathematics, several studies such as Lazzari (2023) and Shukla and McInnis (2021) have shown that flipping the classroom increases their appreciation and love of mathematics.

The large practical importance of the present study's flipped learning intervention could have been contributed by the combination of effective video watching and notetaking, a cognitive strategy proven to magnify students' learning and understanding of content. The evidence also challenges the observations of Cabi (2018), particularly, who found a decrease in students' learning after flipping the classroom. Cabi's (2018) research design, as well as that of other studies with negative results, will need to be re-looked in order to understand why their flipped learning interventions failed to bring about the expected results when the present study was able to produce substantially large gains in student learning as a result of the approach. Factors like educational context, design of the intervention and subject matter might have been at play as moderator variables, as suggested by Bredow et al. (2021), and are worth looking into in future research.

Learning mathematics via the flipped classroom approach was an interesting experience for grade 11 students involved in the study. It was not an approach they were familiar with as the students were taught using didactic, teacher-centered methods for most of their schooling years. In a typical, conventional system, most students would understand or perceive doing mathematics' as being equivalent to listening to teacher talk and watching teacher demonstrations of problem solving. But with the flipped classroom approach, they experienced a constructivist learning environment that empowered them with learner autonomy. They could control the pace of learning and examine the study materials as deeply as they wished without worrying if they were slowing down other learners. The students rely on themselves to learn, and along the way, they discovered that they were the most important part of the teaching-learning process. The students also played a main role in retrieving the required knowledge related to the concept taught throughout watching the videos and by constructing their own lesson notes.

Despite the cognitive challenge and intellectual demand imposed on them by the approach, the students were decidedly happy with the new way of learning and felt that they were "doing better in the subject, than before," as the students expressed in their researcher survey responses after the quasi-experiment was over. Some reported feelings like "more motivated and excited to learn in the class after watching each video" (excerpted from the students' survey responses). Walking into class with a certain degree of prior knowledge on the topic, created a great feeling of confidence in the students. Cumulatively, both the test results and the students' survey responses, confirmed our hypothesis that the flipped classroom model is capable of significantly improving students' performance in mathematics.

Applying the flipped learning approach more consistently in schools, will be very beneficial for all learners. Students who can construct their own understanding of content are more likely to remain motivated and socially engaged.

Limitations of the Study

The study's main limitation was its small sample size. Only 50 students were involved in the quasi-experiment, when ideally, at least 30 subjects (hence a total sample of 60) are required for each group. But to generate statistically more defensible results, between 40 and 50 subjects per group are recommended (Field, 2013). A second limitation was the study did not examine the interaction effect of prior knowledge on the subjects' mathematics performance apart from the main effect of the intervention. Future research should investigate this concern using a more rigorous experimental design, such as a true experimental design using equal groups with at least 40 subjects in each group.

Finally, the study did not explore students' views about which element in the flipped classroom design they found most beneficial to their learning–whether it was the videos, the in-class discussion, or the notetaking that had helped them to acquire the topic's content. Getting students' views on the effectiveness of these elements would have enabled the researchers to understand the results better and will also guide mathematics teachers in designing their flipped instruction.

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Ethical statement: Authors stated that all procedures performed in studies involving human participants is following the ethical standards of the institution, the national research committee, and the 1964 Helsinki Declaration and its later amendments or comparable ethical standards. Thus, the researchers ensured they obtained consent from the school administrations to conduct this experimental study on the assigned students. Also, the consent of all the individual participants included in the study was obtained. Moreover, the study was conducted in compliance with the national ethical guidelines of the Ministry of Education research committee.

Declaration of interest: No conflict of interest is declared by authors.

Data sharing statement: Data supporting the findings and conclusions are available upon request from the corresponding author.

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APPENDIX A: SAMPLE NOTETAKING SHEET

Name____

Section 10.2: Introduction to Conics: Parabolas

Objective: In this lesson you learned how to write the standard form ofthe equation of a parabola.

Important Vocabulary Define each term or concept.

Directrix A fixed line in the plane from which each point on a parabola is the samedistance as the distance from the point to a fixed point in the plane.
Focus A fixed point in the plane from which each point on a parabola is the samedistance as the distance from the point to a fixed line in the plane.
Focal chord A line segment that passes through the focus of a parabola and hasendpoints on the parabola.
Latus rectum the specific focal chord perpendicular to the axis of a parabola.
Tangent A line is tangent to a parabola at a point on the parabola if the line intersects,but does not cross, the parabola at the point.

What you should learnHow to write equations of parabolas in standardform and graph parabolas.

I. Conics (Page 735)
A **conic section,** or **conic,** is the intersection of a planeand a double-napped cone.
Name the four basic conic sections: circle, ellipse, parabola,and hyperbola.
In the formation of the four basic conics, the intersecting planedoes not pass through the vertex of the cone. When the plane does pass through the vertex, the resulting figure is a degenerate conic, such as a point, a line, ora pair of intersecting lines.

What you should learn How to recognize a conicas the intersection of a plane and a double- napped cone

II. Parabolas (Pages 736-738)
A **parabola** is the set of all points (x, y) in a plane thatare equidistant from a fixed line, the directrix, and a fixed point,the focus, not on the line.
The vertex of a parabola is the midpoint between the focus and the directrix.
The axis of the parabola is the line passing through the focus and the vertex.

The standard form of the equation of a parabola with a verticalaxis having a vertex at (h, k) and directrix $y = k - p$ is
$(x - h)^2 = 4p\,(y - k),\ \ p \neq 0$
The standard form of the equation of a parabola with a horizontalaxis having a vertex at (h, k) and directrix $x = h - p$ is:
$(y - k)^2 = 4p\,(x - h)$
The focus lies on the axis p units (directed distance) from the vertex. If the vertex is at the origin (0, 0), the equation takes onone of the following forms:
$x^2 = 4py,\ \ p \neq 0.$ or $y^2 = 4px,\ \ p \neq 0$
Example 1: Find the standard form of the equation of the parabola with vertex at the origin and focus (1, 0).
$y^2 = 4x$

III. Applications of Parabolas (Pages 738 - 739) Describe a real-life situation in which parabolas are used.
Answers will vary.
The reflective property of a parabola states that the tangent line to a parabola at a point P makes equal angles with the followingtwo lines:

1) The line passing through P and the focus
2) The axis of the parabola

What you should learn How to use the reflective property of parabolas to solve real-life problems

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APPENDIX B: PRE-/ POST-TEST

1. Given the parabola: $(y + 1)^2 = 16(x - 3)$. Find
   a. The vertex (1 mark)
   b. The focus (1 mark)
   c. The directrix of the parabola (1 mark)
2. Given the ellipse: $\frac{(x-2)^2}{9} + \frac{(y-1)^2}{4} = 1$. Find
   a. The center (1 mark)
   b. The vertices (4 marks)
   c. The foci (2 marks)
   d. The eccentricity (2 marks)
3. Given the hyperbola: $\frac{(x+1)^2}{9} - \frac{(y+1)^2}{4} = 1$. Find
   a. The center (1 mark)
   b. The foci (2 marks)
   c. The asymptotes (2 marks)
4. Use completing square to determine whether the equation is an equation of parabola, ellipse or a hyperbola: $x^2 - 5y^2 - 2x + 20y = 44$ (3 marks)

APPENDIX C: PRE-TEST / POST-TEST RESULTS

Student No.	Control group Mark out of 20	Experimental group Mark out of 20
1	13	\
2	1	1
3	11	2
4	3	1
5	5	4
6	1	6
7	10	5
8	1	3
9	4	2
10	14	9
11	13	5
12	4	3
13	16	6
14	9	12
15	3	2
16	5	2
17	5	7
18	5	6
19	2	2
20	2	8
21	1	2
22	3	8
23	12	8
24	13	12
25	8	5

Pre-test Results for Control and Experimental Groups

Student No	Control group Mark out of 20	Experimental group Mark out of 20
1	16	17.5
2	14	13
3	15	19.5
4	16	17
5	11	13
6	5	19
7	15	16
8	3	20
9	12	15
10	17	13.5
11	10.5	15
12	12	15.5
13	14	16
14	13	14
15	16	10
16	15.5	13
17	6	20
18	20	18
19	18	12
20	2	20
21	1	17
22	3	20
23	13.5	15
24	17.5	17
25	4.5	15

Post-test Results for Control and Experimental Groups